Eratosthenes was a Greek mathematician who lived from about 276 B.C. to 194 B.C. He devised the **Sieve of Eratosthenes** as a method of identifying all the prime numbers up to a certain number. Using the chart below, you can use his method to find all the prime numbers up to 120. Just follow these numbered steps.

1. The number 1 is not prime. Cross it out.

2. The number 2 is prime. Circle it. Then cross out every second number—4, 6, 8, 10, and so on.

3. The number 3 is prime. Circle it. Then cross out every third number—6, 9, 12, and so on.

4. The number 4 is crossed out. Go to the next number that is not crossed out.

5. The number 5 is prime. Circle it. Then cross out every fifth number—10, 15, 20, 25, and so on.

6. Continue crossing out numbers as described in Steps 2–5. The numbers that remain at the end of this process are prime numbers.

7. **CHALLENGE** Look at the prime numbers that are circled in the chart. Do you see a pattern among the prime numbers that are greater than 3? What do you think the pattern is?

<table>
<thead>
<tr>
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<th>1</th>
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<td>115</td>
<td>116</td>
<td>117</td>
<td>118</td>
<td>119</td>
<td>120</td>
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</tbody>
</table>
**Enrich**

**Making Models for Numbers**

Have you wondered why we read the number $3^2$ as three squared? The reason is that a common model for $3^2$ is a square with sides of length 3 units. As you see, the figure that results is made up of 9 square units.

Make a model for each expression.

1. $2^2$
2. $4^2$
3. $1^2$
4. $5^2$

Since we read the expression $2^3$ as *two cubed*, you probably have guessed that there is also a model for this number. The model, shown at the right, is a cube with sides of length "2 units. The figure that results is made up of 8 cubic units.

Exercises 5 and 6 refer to the figure to the right.

5. What expression is being modeled? _____

6. Suppose that the entire cube is painted red. Then the cube is cut into small cubes along the lines shown.
   a. How many small cubes are there in all? _____
   b. How many small cubes have red paint on exactly three of their faces? _____
   c. How many small cubes have red paint on exactly two of their faces? _____
   d. How many small cubes have red paint on exactly one face? _____
   e. How many small cubes have no red paint at all? _____

7. **CHALLENGE** In the space at the right, draw a model for the expression $4^3$. 
Now that you have learned how to evaluate an expression using the order of operations, can you work backward? In this activity, the value of the expression will be given to you. It is your job to decide what the operations or the numbers must be in order to arrive at that value.

**Fill in each □ with +, −, ×, or ÷ to make a true statement.**

1. 48 □ 3 □ 12 = 12
2. 30 □ 15 □ 3 = 6
3. 24 □ 12 □ 6 □ 3 = 4
4. 24 = □ 12 □ 6 □ 3 = 18
5. 4 □ 16 □ 2 □ 8 = 24
6. 45 □ 3 □ 3 □ 9 = 3
7. 36 □ 2 □ 3 □ 12 □ 2 = 0
8. 72 □ 12 □ 4 □ 8 □ 3 = 0

**Fill in each □ with one of the given numbers to make a true statement. Each number may be used only once.**

9. 6, 12, 24

□ ÷ □ × □ = 12

10. 4, 9, 36

□ − □ ÷ □ = 0

11. 6, 8, 12, 24

□ ÷ □ + □ − □ = 4

12. 2, 5, 10, 50

□ − □ ÷ □ + □ = 50

13. 2, 4, 6, 8, 10

□ ÷ □ × □ + □ − □ = 0

14. 1, 3, 5, 7, 9

□ ÷ □ × □ − □ ÷ □ = 1
There are many times when you need to make an estimate in relation to a reference point. For example, at the right there are prices listed for some school supplies. You might wonder if $5 is enough money to buy a small spiral notebook and a pen. This is how you might estimate, using $5 as the reference point.

- The notebook costs $1.59 and the pen costs $3.69.
- $1 + $3 = $4. I have $5 - $4, or $1, left.
- $0.59 and $0.69 are each more than $0.50, so $0.59 + $0.69 is more than $1.

So, $5 will not be enough money.

Use the prices at the right to answer each question.

1. Jamaal has $5. Will that be enough money to buy a large spiral notebook and a pack of pencils?

2. Andreas wants to buy a three-ring binder and two packs of filler paper. Will $7 be enough money?

3. Rosita has $10. Can she buy a large spiral notebook and a pen and still have $5 left?

4. Kevin has $10 and has to buy a pen and two small spiral notebooks. Will he have $2.50 left to buy lunch?

5. What is the greatest number of erasers you can buy with $2?

6. What is the greatest amount of filler paper that you can buy with $5?

7. Select five items whose total cost is as close as possible to $10, but not more than $10.
You can use variables and expressions to describe patterns. These tile letters grow according to different patterns.

Find a rule that will tell how many tiles it takes to build any size of the letter I.

1. Look for a pattern. Describe the pattern using your own words.

2. Describe the pattern for I using variables and expressions.

Use the rule to predict the number of tiles needed for each I.

3. size 12 ________
4. size 15 ________
5. size 22 ________
6. size 100 ________

7. Suppose you had 39 tiles. What is the largest size I you could make?

8. Find the pattern for the letter X. How many tiles are needed for size 16 of letter X?
Function rules are often used to describe geometric patterns. In the pattern at the right, for example, do you see this relationship?

1st figure: \(3 \times 1 = 3\) dots  
2nd figure: \(3 \times 2 = 6\) dots  
3rd figure: \(3 \times 3 = 9\) dots  
4th figure: \(3 \times 4 = 12\) dots

So the “"nth”" figure in this pattern would have \(3 \times n\), or \(3n\), dots. A function rule that describes the pattern is \(3n\).

**Write a function rule to describe each dot pattern.**

1.  
2.  
3.  
4.  
5.  
6.
Check whether each answer is reasonable. Explain.

1. Margo’s fastest rate in her hot-air balloon was 30 miles per hour. She calculates that if she can fly at the same rate, she can travel 145 miles in about 5 hours. Is her answer reasonable? Explain.

2. Noboru is entering a hot-air balloon race. The race covers 210 miles. He wants to finish the race in 8 hours. He calculates that he must travel at an average rate of 40 miles per hour. Is his answer reasonable? Explain.

3. Olga must meet her brother at the end of the balloon race. The race is 120 miles and the record for the race is 23 miles per hour. She decides that she should be at the finish line 12 hours after the race begins. Is her answer reasonable? Explain.

4. Adam is starting a business to take people on hot-air balloon rides. He knows that to carry 2 people, the balloon must have a volume of about 60,000 cubic feet. For his business, he wants a balloon that will carry 4 people. He calculates that the balloon must have a volume of 120,000 cubic feet. Is his answer reasonable? Explain.

5. Marta’s family is planning a trip through the mountains. They expect to travel 210 miles in about $5\frac{1}{4}$ hours on the mountain roads. Marta calculates that they will be traveling about 100 miles per hour. Is her answer reasonable? Explain.
In an equation chain, you use the solution of one equation to help you find the solution of the next equation in the chain. The last equation in the chain is used to check that you have solved the entire chain correctly.

Complete each equation chain.

1. \(5 + a = 12\), so \(a = \) _______.
   \(ab = 14\), so \(b = \) _______.
   \(16 ÷ b = c\), so \(c = \) _______.
   \(14 - d = c\), so \(d = \) _______.
   \(e ÷ d = 3\), so \(e = \) _______.
   \(a + e = 25 \leftarrow \text{Check:} \)

2. \(9f = 36\), so \(f = \) _______.
   \(g = 13 - f\), so \(g = \) _______.
   \(63 ÷ g = h\), so \(h = \) _______.
   \(h + i = 18\), so \(i = \) _______.
   \(j - i = 9\), so \(j = \) _______.
   \(j ÷ f = 5 \leftarrow \text{Check:} \)

3. \(m ÷ 4 = 8\), so \(m = \) _______.
   \(m - n = 12\), so \(n = \) _______.
   \(np = 100\), so \(p = \) _______.
   \(q = 40 + p\), so \(q = \) _______.
   \(p + q - 10 = r\), so \(r = \) _______.
   \(r - m = 8 \leftarrow \text{Check:} \)

4. \(18 = v - 12\), so \(v = \) _______.
   \(v ÷ w = 3\), so \(w = \) _______.
   \(80 = wx\), so \(x = \) _______.
   \(w + x = 2y\), so \(y = \) _______.
   \(xy - z = 40\), so \(z = \) _______.
   \(z - v = 2 \leftarrow \text{Check:} \)

5. Create your own equation chain using these numbers for the variables: \(a = 10\), \(b = 6\), \(c = 18\), and \(d = 3\).
The figure at the right is the floor plan of a family room. The plan is drawn on grid paper, and each square of the grid represents one square foot. The floor is going to be covered completely with tiles.

1. What is the area of the floor?

2. Suppose each tile is a square with a side that measures one foot. How many tiles will be needed?

3. Suppose each tile is a square with a side that measures one inch. How many tiles will be needed?

4. Suppose each tile is a square with a side that measures six inches. How many tiles will be needed?

Use the given information to find the total cost of tiles for the floor.

5. tile: square, 1 foot by 1 foot cost of one tile: $3.50

6. tile: square, 6 inches by 6 inches cost of one tile: $0.95

7. tile: square, 4 inches by 4 inches cost of one tile: $0.50

8. tile: square, 2 feet by 2 feet cost of one tile: $12

9. tile: square, 1 foot by 1 foot cost of two tiles: $6.99

10. tile: rectangle, 1 foot by 2 feet cost of one tile: $7.99

11. Refer to your answers in Exercises 5–10. Which way of tiling the floor costs the least? the most?
Six students took a music survey in their school. They tallied the results in the table below.

<table>
<thead>
<tr>
<th>Favorite Types of Music Survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandra’s Result</td>
</tr>
<tr>
<td>Rap 12</td>
</tr>
<tr>
<td>Rock 5</td>
</tr>
<tr>
<td>Pop 9</td>
</tr>
<tr>
<td>Arlan’s Results</td>
</tr>
<tr>
<td>Rap 6</td>
</tr>
<tr>
<td>Pop 14</td>
</tr>
<tr>
<td>Country 3</td>
</tr>
<tr>
<td>Juan’s Results</td>
</tr>
<tr>
<td>Jazz 2</td>
</tr>
<tr>
<td>Rap 7</td>
</tr>
<tr>
<td>Pop 3</td>
</tr>
<tr>
<td>Classical 1</td>
</tr>
<tr>
<td>Erin’s Results</td>
</tr>
<tr>
<td>Rock 8</td>
</tr>
<tr>
<td>Country 4</td>
</tr>
<tr>
<td>Blues 3</td>
</tr>
<tr>
<td>Pop 8</td>
</tr>
<tr>
<td>Trevor’s Results</td>
</tr>
<tr>
<td>Rap 15</td>
</tr>
<tr>
<td>Jazz 4</td>
</tr>
<tr>
<td>Pop 11</td>
</tr>
<tr>
<td>Anya’s Results</td>
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<tr>
<td>Rock 4</td>
</tr>
<tr>
<td>Country 6</td>
</tr>
<tr>
<td>Pop 2</td>
</tr>
</tbody>
</table>

1. The students decide to combine the results of their survey and display them in a graph. They want no more than 5 categories represented. How can they group their data to best reflect the results within a limit of 5 categories?

2. Make a table to show the data in 5 categories.

3. Which type of graph would you use to display the data in the table you made? Explain. Then make a graph.
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*Interpret Line Graphs*

You can find your heartbeat by taking your pulse. To feel your pulse, use your forefinger (pointer), but not your thumb, on either an artery on your neck or on the underside of your wrist.

- Work with a partner. Use a watch with a second hand and count your pulse for exactly one minute.
- Record your pulse four times as indicated in the table.
- Using graph paper, make a double-line graph that shows both partners’ heartbeats.

<table>
<thead>
<tr>
<th>Heartbeat</th>
<th>You</th>
<th>Partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>At rest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>After 2 minutes of exercise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>After 1 minute rest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>After 2 minutes rest</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answer these questions based on the double-line graph you and your partner made.**

1. Describe the difference between your heartbeat at rest and after exercising.

__________________________________________________________________________

__________________________________________________________________________

2. Describe the difference between your partner’s heartbeat at rest and after 2 minutes of exercising.

__________________________________________________________________________

3. Find the average heartbeat per minute for you and your partner when you are at rest. (Round to the nearest whole number.)

__________________________________________________________________________

4. Write your own question based on the double-line graph. Have your partner answer the question.

__________________________________________________________________________
Choosing a Sample

Statisticians often use samples to represent larger groups. For example, television ratings are based on the opinions of a few people who are surveyed about a program. When using samples, people taking surveys must make sure that their samples are representative of the larger group in order to ensure that their conclusions are not misleading.

A company that makes athletic shoes is considering hiring a professional basketball player to appear in its commercials. Before hiring him, they are doing research to see if he is popular with teens. Would they get good survey results from taking a survey about the basketball player from each of these surveys?

1. 200 teens at a basketball game of the basketball player’s team

2. 25 teens at a shopping mall

3. 500 students at a number of different middle and high schools

Decide whether each location is a good place to find a representative sample for the selected survey. Justify your answer.

4. number of hours of television watched in a month at a shopping mall

5. favorite kind of entertainment at a movie theater
In a **line plot**, data are pictured on a number line. An X is used to represent each item of data. For example, the figure below is a line plot that pictures data about the number of CDs owned by the students in a math class.

**Number of CDs Owned by Students in a Math Class**

Use the line plot above to answer each question.

1. How many students own exactly eighteen CDs? ____________

2. What number of CDs is owned by exactly three students? ______

3. A data item that is far apart from the rest of the data is called an **outlier**. Is there an outlier among these data? What is it? ______

4. What would you say is the number of CDs owned by the “typical” student in this class? __________________________

5. Use the data in the table to complete the line plot below. Four data points have been graphed for you.

<table>
<thead>
<tr>
<th>Number of Seconds for 24 Sixth-Graders to Run 200 Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>130 100 85 120 100 100 110 150 90 100 110 130</td>
</tr>
<tr>
<td>125 105 100  70 125 85 95 130 105  90 105 100</td>
</tr>
</tbody>
</table>

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Make a table to determine how much of a certain item is needed for a pool party you are having. Assume that 10 people will attend your party, and that in addition to a sandwich and a drink for everyone, you will also need to provide pool supplies for some of your guests.

After you have created your table, answer the following questions:

1. What item will most of your guests need you to provide?

2. What item will the least number of guests need you to provide?

3. Explain how making your table might help you to plan better for your pool party.
A line plot is a version of a bar graph. Look at the line plot on the right. It shows the results of a survey about TV viewing habits. Twenty-two students were asked how many hours of television they watch in one week. Three students said they watch 2 hours of television each week. Two students said they watch 9 hours of television per week.

A bar graph is another way to display data. You can use a line plot to create a bar graph.

Change each line plot into a bar graph.

1. **Number of Siblings Students Have**

2. **Class Scores on a Math Quiz**
Mean, median and mode are all different types of averages. Average is not a mathematical word. In mathematics, it is necessary to specify which type of average you are using.

1. The prices of seven homes for sale in Sunnydale are $151,000; $148,500; $163,000; $180,500; $151,000; $172,000; $189,000. Find the mean, median, and mode for the price of the homes for sale.

2. A real estate agent is writing an advertisement for a newspaper. She writes, “The average price of a home in Sunnydale is $151,000.” Which average did she use? Why did she choose it? Is it misleading?

3. Which type of average should be used to best represent the “average” price of a home in Sunnydale?

A bead company is having a special promotion for which it includes special blue-colored beads in its packages. The line plot shows how many blue beads were found in each of 19 packages.

Sam, Matt, and Carla solve for the average number of blue beads per package. Sam’s average has the highest value, then Carla, and then Matt’s. Their teacher tells them that each one has a correct answer.

4. Determine which average each student found and what it is.
Each puzzle on this page contains an incomplete set of data. The clues give you information about the mean, median, mode, or range of the data. Working from these clues, you can decide what the missing data items must be. For example, this is how you might solve the data puzzle at the right.

There are 6 items of data.
The mean is 18, so the sum of the data must be $6 \times 18 = 108$.
Add the given data: $12 + 17 + 18 + 19 + 19 = 85$.
So the complete set of data is: 12, 17, 18, 19, 19, 23.

Find the missing data. (Assume that the data items are listed in order from least to greatest.)

1. **Clue:** mode = 8
   
   **Data:** 7, 7, 8, __, __, 14

2. **Clue:** median = 54.5
   
   **Data:** 36, 40, 49, __, 65, 84

3. **Clues:** mean = 27
   
   mode = 30
   
   **Data:** 10, 25, 27, __, 30, __

4. **Clues:** median = 120
   
   range = 46
   
   **Data:** 110, 112, __, 124, 136, __

5. **Clues:** mean = 13
   
   median = 13
   
   range = 13
   
   **Data:** __, 9, 12, __, 18, __

6. **Clues:** mean = 7
   
   median = 8.5
   
   mode = 10
   
   **Data:** __, 4, 8, __, __, __

7. **Clues:** mean = 60
   
   mode = 52
   
   range = 28
   
   **Data:** __, 52, __, __, 72, 78

8. **Clues:** median = 24
   
   mode = 28
   
   range = 24
   
   **Data:** 6, 15, __, __, __, __
Deciding what type of graph to use is just as important as knowing how to make a graph. Here are guidelines to help you make that decision.

- A **bar graph** compares data that fall into distinct categories, such as the populations of several cities compare in one year.

- A **line graph** shows changes in data over a period of time, such as the population of one city changing over several years.

- A **histogram** uses bars to represent the frequency of numerical data organized in intervals.

Make an appropriate graph for each set of data.

1. **Taxis in Use**

<table>
<thead>
<tr>
<th>Year</th>
<th>Number (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>135</td>
</tr>
<tr>
<td>2000</td>
<td>136</td>
</tr>
<tr>
<td>2001</td>
<td>142</td>
</tr>
<tr>
<td>2002</td>
<td>148</td>
</tr>
</tbody>
</table>

2. **Aircraft Capacity**

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>B747</td>
<td>405</td>
</tr>
<tr>
<td>DC-10</td>
<td>288</td>
</tr>
<tr>
<td>L-1011</td>
<td>296</td>
</tr>
<tr>
<td>MD-80</td>
<td>142</td>
</tr>
</tbody>
</table>

3. **Video Games Owned**

<table>
<thead>
<tr>
<th>Number of Games</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>5</td>
</tr>
<tr>
<td>3–5</td>
<td>4</td>
</tr>
<tr>
<td>6–8</td>
<td>9</td>
</tr>
<tr>
<td>9–11</td>
<td>6</td>
</tr>
</tbody>
</table>
Enrich
Graphs with Integers

Statistical graphs that display temperatures, elevations, and similar data often involve negative quantities. On graphs like these, the scale usually will have a zero point and will include both positive and negative numbers.

For Exercises 1–6, use the bar graph at the right to answer each question.

1. In which cities is the record low temperature greater than 0°F?

2. In which cities is the record low temperature less than 0°F?

3. In which city is the record low temperature about −25°F?

4. Estimate the record low temperature for New York City.

5. In which cities is the record low temperature less than twenty degrees from 0°F?

6. How many degrees are between the record low temperatures for Bismarck and Honolulu?

7. In the space at the right, make a bar graph for the data below.

Altitudes of Some California Locations Relative to Sea Level

<table>
<thead>
<tr>
<th>Location</th>
<th>Altitude (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alameda</td>
<td>30</td>
</tr>
<tr>
<td>Brawley</td>
<td>−112</td>
</tr>
<tr>
<td>Calexico</td>
<td>7</td>
</tr>
<tr>
<td>Death Valley</td>
<td>−282</td>
</tr>
<tr>
<td>El Centro</td>
<td>−39</td>
</tr>
<tr>
<td>Salton City</td>
<td>−230</td>
</tr>
</tbody>
</table>
The letter A at the right was created by shading part of a hundreds square. There are 26 parts shaded, so the value of the letter A is 26 hundredths, or 0.26.

Find the value of each letter.

1. 2. 3. 4. 5.

6. 7. 8. 9. 10.


16. 17. 18. 19. 20.

21. 22. 23. 24. 25.
Enrich

A Look at Nutrients

The table below gives data about a few of the nutrients in an average serving of some common foods.

<table>
<thead>
<tr>
<th>Food</th>
<th>Protein (grams)</th>
<th>Fat (grams)</th>
<th>Carbohydrates (grams)</th>
<th>Vitamins (milligrams)</th>
<th>Minerals* (milligrams)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>apple (medium)</td>
<td>0.3</td>
<td>0.5</td>
<td>21.1</td>
<td>8</td>
<td>0.02 0.02 1 159 10</td>
</tr>
<tr>
<td>chocolate bar (1.02 oz)</td>
<td>2.2</td>
<td>9.4</td>
<td>16.5</td>
<td>0</td>
<td>0.02 0.08 29 119 55</td>
</tr>
<tr>
<td>cola (12 fl oz)</td>
<td>0.0</td>
<td>0.0</td>
<td>40.7</td>
<td>0</td>
<td>0.00 0.00 20 7 11</td>
</tr>
<tr>
<td>hamburger (1 medium)</td>
<td>21.8</td>
<td>14.5</td>
<td>0.0</td>
<td>0</td>
<td>0.13 0.15 40 382 6</td>
</tr>
<tr>
<td>orange juice (8 fl oz)</td>
<td>1.7</td>
<td>0.1</td>
<td>26.8</td>
<td>97</td>
<td>0.20 0.05 2 474 22</td>
</tr>
<tr>
<td>peas (1/2 cup)</td>
<td>4.5</td>
<td>0.4</td>
<td>10.8</td>
<td>19</td>
<td>0.22 0.09 128 137 17</td>
</tr>
<tr>
<td>wheat bread (1 slice)</td>
<td>2.3</td>
<td>1.0</td>
<td>11.3</td>
<td>0</td>
<td>0.11 0.08 129 33 30</td>
</tr>
<tr>
<td>whole milk (8 fl oz)</td>
<td>8.0</td>
<td>8.2</td>
<td>11.4</td>
<td>2</td>
<td>0.09 0.40 120 370 291</td>
</tr>
</tbody>
</table>

*Na = sodium, K = potassium, Ca = calcium

Use the data in the table to answer each question.

1. Is there more potassium in one apple or in one serving of peas? ________________

2. Does one serving of milk contain more fat or more carbohydrates? ________________

3. Which foods contain less than 0.05 milligram of vitamin B-2? ________________

4. Which foods contain an amount of carbohydrates between 15 grams and 25 grams? ________________

5. Which food contains the least amount of calcium? ________________

6. List the foods in order of their protein content from least to greatest. ________________
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Everybody into the Pool!

Answer each question using the “decimal pool” below.

1. Which decimal when rounded to the nearest hundredth is 0.03? _____________
2. Which decimal when rounded to the nearest thousandth is 0.003? _____________
3. Which two decimals when rounded to the nearest hundredth are 0.02? _____________
4. Which five decimals when rounded to the nearest tenth are 0.2? _____________
5. Which decimal when rounded to the nearest thousandth is 0.210? _____________
6. Which two decimals when rounded to the nearest hundredth are 0.20? _____________
7. Add to the pool four different decimals that when rounded to the nearest thousandth are 0.301.

8. Add to the pool a three-place decimal that when rounded to the nearest tenth is 1.0.

9. **CHALLENGE** Suppose that you are rounding decimals to the nearest hundredth. How many three-place decimals round to 0.05? List them.
One morning, Sasha, Daria, and Lucas were in such a hurry to get to school, they bumped into each other and dropped all their backpacks and lunches. They picked up their belongings and rushed to class, but at lunchtime, they discovered they had all picked up the wrong brown bag. Sasha got the bag with a peanut butter sandwich and chocolate milk. Daria’s bag contained a ham sandwich and strawberry yogurt shake. Lucas ended up with a cucumber sandwich and carrot sticks.

1. Using logical reasoning, explain how you would return the lunches to their original owners. You have the following clues:
   - Lucas won’t eat fruits or vegetables.
   - Sasha loves strawberries and eats them every day.
   - Daria is allergic to milk.

<table>
<thead>
<tr>
<th></th>
<th>PB and CM</th>
<th>HS and SYS</th>
<th>CS and CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sasha</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daria</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lucas</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write your own “mystery” and give it to a friend to solve.
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Horizontal Estimation

Many times an addition problem is given to you in horizontal form, with the addends written from left to right. To estimate the sum, you don’t have to rewrite the addition vertically in order to line up the decimal points. Just use place value to figure out which digits are most important. Here is an example:

3.11 + 0.4639 + 8.205

The most important digits are in the ones place.
3 + 0 + 8 = 11

The next group of important digits are in the tenths place.
1 tenth + 4 tenths + 2 tenths = 7 tenths

Add to make your estimate: 11 + 7 tenths → about 11.7

Estimate each sum.

1. 7.44 + 0.2193 __________
2. 0.4015 + 9.3 + 3.264 __________
3. 0.4208 + 0.16 __________
4. 0.52 + 0.1 + 0.308 + 0.0294 __________
5. 10.2 + 0.519 __________
6. 12.004 + 1.5 + 4.32 + 0.1009 __________
7. 6.72 + 0.5037 __________
8. 0.805 + 1.006 + 0.4 + 2.0305 __________
9. 1.208 + 3.1 + 0.04 + 6.143 + 0.3075 __________
10. 0.9005 + 5.03 + 7.108 + 0.004 + 10.7 __________

This same method works when you need to estimate a sum of much greater numbers. Estimate each sum.

11. 53,129 + 420,916 __________
12. 6,048 + 2,137 + 509 __________
13. 723 + 4,106 + 4,051 + 318 __________
14. 7,095 + 12,402 + 3,114 + 360 __________
15. 650,129 + 22,018 + 107,664 + 10,509 __________
Would you give an estimate or an exact answer? Explain, then solve.

1. Suppose a mint make 2,300,000 nickels per day. At that rate, can they make at least 60,000,000 nickels in 32 days?

2. A private coin company makes commemorative coins of famous singers. The coins are sealed in individual boxes. The total weight of the coin and the box is 4 ounces. The individual boxes are packed in large boxes. How many coins are there in a large box that weighs 45 pounds?

3. A commemorative coin company has 39,485,500 coins for sale. If 11,890,500 coins have already been sold, about how many coins does the company still have in stock?

4. A magazine has a circulation of 18,000,000. Suppose 5,000 magazines are printed every hour for 3 days. Will the magazine print enough magazines in 3 days to cover their circulation?

5. The circulation of a magazine has increased by 5,325,000 from last year. The circulation was 12,937,500 last year. What is the new circulation of the magazine?

6. It costs $0.25 to mail a magazine. If there are 3,480,000 magazines, will $900,000 cover the postage?
The currency used in the United States is the U.S. dollar. Each dollar is divided into 100 cents. Most countries have their own currencies. On January 1, 2002, 12 countries in Europe converted to a common monetary unit that is called the euro.

The symbol, €, is used to indicate the euro.

The exchange rate between dollars and euros changes every day. $1.00 is worth about 0.85€.

**EXERCISES Add or subtract to solve each problem.**

1. Henry bought a pair of shoes for €34.75 and a pair of pants for €21.49. How much money did he spend? ____________

2. Louis receives €10.50 a week for doing his chores. His sister is younger and has fewer chores. She receives €5.25. How much money do Louis and his sister receive together in one week? ____________

3. A gallon of Brand A of vanilla ice cream costs €5.49. A gallon of Brand B vanilla ice cream costs €4.87. How much money will Luca save if he buys Brand A instead of Brand B? ____________

4. Michael passed up a pair of jeans that cost €29.50 and decided to buy a pair that were only €15.86. How much money did he save by buying the less expensive jeans? ____________

5. Jesse’s favorite magazine costs €1.75 at the store. If he buys a subscription, each issue is only 0.37€. How much money will Jesse save on each issue if he buys a subscription? ____________

6. Layla wants to buy a CD for €11.99 and a book for €6.29. She has €15.00. How much more money does she need to buy both the CD and the book? ____________

7. **CHALLENGE** Lynne’s lunch came to €4.00. Her drink was €1.50. How much did she spend total? What would be the equivalent dollar amount? ____________

8. **CHALLENGE** At the grocery store, Jaden purchased a box of cereal for $3.55 and a gallon of milk for $2.89. He gave the cashier $10.00. How much change did he receive? What would be the equivalent euro amount?
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GCFs by Successive Division

Here is a different way to find the greatest common factor (GCF) of two numbers. This method works well for large numbers.

Find the GCF of 848 and 1,325.

**Step 1**
Divide the smaller number into the larger.

\[
\begin{array}{c}
848)1,325 \\
848 \\
477
\end{array}
\]

**Step 2**
Divide the remainder into the divisor. Repeat this step until you get a remainder of 0.

\[
\begin{array}{c}
477)848 \\
371)477 \\
106)371 \\
53)106
\end{array}
\]

**Step 3**
The last divisor is the GCF of the two original numbers.
The GCF of 848 and 1,325 is 53.

Use the method above to find the GCF for each pair of numbers.

1. 187; 578
2. 161; 943
3. 215; 1,849
4. 453; 484
5. 432; 588
6. 279; 403
7. 1,325; 3,498
8. 9,840; 1,751
9. 3,484; 5,963
10. 1,802; 106
11. 45,787; 69,875
12. 35,811; 102,070
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LCD Fraction Riddles

A conjecture is an educated guess or an opinion. Mathematicians and scientists often make conjectures when they observe patterns in a collection of data. On this page, you will be asked to make a conjecture about polygons.

Use a protractor to measure the angles of each polygon. Then find the sum of the measures. (Use the quadrilateral at the right as an example.)

1. 

2. 

3. 

4. 

5. Make a conjecture. How is the sum of the angle measures of a polygon related to the number of sides?

6. Test your conjecture. On a clean sheet of paper, use a straightedge to draw a hexagon. What do you guess is the sum of the angle measures? Measure each angle and find the sum. Was your conjecture true?
Fraction Mysteries

Here is a set of mysteries that will help you sharpen your thinking skills. In each exercise, use the clues to discover the identity of the mystery fraction.

1. My numerator is 6 less than my denominator.
   I am equivalent to $\frac{3}{4}$.

2. My denominator is 5 more than twice my numerator.
   I am equivalent to $\frac{1}{3}$.

3. The GCF of my numerator and denominator is 3.
   I am equivalent to $\frac{2}{5}$.

4. The GCF of my numerator and denominator is 5.
   I am equivalent to $\frac{4}{6}$.

5. My numerator and denominator are prime numbers.
   My numerator is one less than my denominator.

6. My numerator and denominator are prime numbers.
   The sum of my numerator and denominator is 24.

7. My numerator is divisible by 3.
   My denominator is divisible by 5.
   My denominator is 4 less than twice my numerator.

8. My numerator is divisible by 3.
   My denominator is divisible by 5.
   My denominator is 3 more than twice my numerator.

9. My numerator is a one-digit prime number.
   My denominator is a one-digit composite number.
   I am equivalent to $\frac{8}{32}$.

10. My numerator is a prime number.
    The GCF of my numerator and denominator is 2.
    I am equivalent to $\frac{1}{5}$.

11. **CHALLENGE** Make up your own mystery like the ones above. Be sure that there is only one solution. To check, have a classmate solve your mystery.
It is common to see mixed fractions in recipes. A recipe for a pizza crust may ask for \(1\frac{1}{2}\) cups of flour. You could measure this amount in two ways. You could fill a one-cup measuring cup with flour and a one-half-cup measuring cup with flour or you could fill a half-cup measuring cup three times, because \(1\frac{1}{2}\) is the same as \(\frac{3}{2}\).

In the following recipes, some mixed numbers have been changed to improper fractions and other fractions may not be written in simplest form. Rewrite each recipe as you would expect to find it in a cookbook.

<table>
<thead>
<tr>
<th>Quick Pizza Crust</th>
<th>Apple Crunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3}{2}) cups flour</td>
<td>(\frac{3}{2}) cups white sugar</td>
</tr>
<tr>
<td>(\frac{2}{4}) cup water</td>
<td>(\frac{3}{2}) cups brown sugar</td>
</tr>
<tr>
<td>(\frac{9}{4}) teaspoons yeast</td>
<td>(\frac{4}{2}) cups of flour</td>
</tr>
<tr>
<td>(\frac{2}{2}) teaspoon salt</td>
<td>(\frac{4}{2}) cups oatmeal</td>
</tr>
<tr>
<td>(\frac{4}{4}) teaspoon sugar</td>
<td>(\frac{8}{3}) sticks margarine</td>
</tr>
<tr>
<td>(\frac{8}{8}) tablespoon oil</td>
<td>(\frac{2}{2}) teaspoon salt</td>
</tr>
</tbody>
</table>
If someone asked you to name a fraction between $\frac{4}{7}$ and $\frac{6}{7}$, you would probably give the answer $\frac{5}{7}$ pretty quickly. But what if you were asked to name a fraction between $\frac{4}{7}$ and $\frac{5}{7}$? At the right, you can see how to approach the problem using “fraction sense.” So, one fraction between $\frac{4}{7}$ and $\frac{5}{7}$ is $\frac{9}{14}$.

Use your fraction sense to solve each problem.

1. Name a fraction between $\frac{1}{3}$ and $\frac{2}{3}$. ______
2. Name a fraction between $\frac{3}{5}$ and $\frac{4}{5}$. ______
3. Name five fractions between $\frac{1}{2}$ and 1. ______
4. Name five fractions between 0 and $\frac{1}{4}$. ______
5. Name a fraction between $\frac{1}{4}$ and $\frac{1}{2}$ whose denominator is 16. ______
6. Name a fraction between $\frac{2}{3}$ and $\frac{3}{4}$ whose denominator is 10. ______
7. Name a fraction between 0 and $\frac{1}{6}$ whose numerator is 1. ______
8. Name a fraction between 0 and $\frac{1}{10}$ whose numerator is not 1. ______
9. Name a fraction that is halfway between $\frac{2}{9}$ and $\frac{5}{9}$. ______
10. Name a fraction between $\frac{1}{4}$ and $\frac{3}{4}$ that is closer to $\frac{1}{4}$ than $\frac{3}{4}$. ______
11. Name a fraction between 0 and $\frac{1}{2}$ that is less than $\frac{3}{10}$. ______
12. Name a fraction between $\frac{1}{2}$ and 1 that is less than $\frac{3}{5}$. ______
13. Name a fraction between $\frac{1}{2}$ and $\frac{3}{4}$ that is greater than $\frac{4}{5}$. ______
14. How many fractions are there between $\frac{1}{4}$ and $\frac{1}{2}$? ______
You can write a decimal or a fraction for the shaded part of any 10-by-10 grid picture. Try to find a pattern in each grid to help you count the number of shaded squares. Write a decimal for the shaded part of each grid picture. Then write a fraction in simplest form that is equivalent to the decimal.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

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Use a Diagram

Draw a diagram to solve.

1. A window design is made of a rectangle divided by two diagonals. How many sections are there and what are their shapes?

2. Sandra draws a regular hexagon. She divides the hexagon into sections by drawing a line from one vertex of the hexagon to the opposite vertex. How many sections are there and what are their shapes?

3. Harold divides a triangle into sections by drawing a line from one vertex of the triangle to the center of the opposite line. How many sections are there and what are their shapes?

4. A tile is shaped like a hexagon. A design on the tile uses 3 lines to divide the hexagon into sections by connecting all the opposite vertices on the hexagon. How many sections are there and what are their shapes?

5. A student divides a pentagon into sections by drawing a line from one vertex to the center of the opposite line. How many sections are there and what are their shapes?
Enrich

Fractional Estimates

Often you only need to give a fractional estimate for a decimal. To make fractional estimates, it helps to become familiar with the fraction-decimal equivalents shown in the chart at the right. You also should be able to identify the fraction as an overestimate or underestimate. Here’s how.

The decimal 0.789 is a little less than 0.8, so it is a little less than $\frac{4}{5}$. Write $\frac{4}{5}$.

The decimal 1.13 is a little more than 1.125, so it is a little more than $1\frac{1}{8}$. Write $1\frac{1}{8}$.

Write a fractional estimate for each decimal. Be sure to identify your estimate as an overestimate or an underestimate.

1. 0.243  
2. 0.509  
3. 0.429

4. 0.741  
5. 0.88  
6. 0.63

7. 0.09  
8. 0.57  
9. 1.471

10. 2.76  
11. 1.289  
12. 5.218

13. The scale in the delicatessen shows 0.73 pound. Write a fractional estimate for this weight.

14. Darnell ordered a quarter pound of cheese. The scale shows 0.23 pound. Is this more or less than he ordered?

15. On the stock market, prices are listed as halves, fourths, and eighths of a dollar. Yesterday the price of one share of a stock was $25.61. Write a fractional estimate for this amount.

16. Charlotte used a calculator to figure out how many yards of ribbon she needed for a craft project. The display shows 2.53125. Write a fractional estimate for this length.
Which of \( \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \) and \( \frac{9}{10} \) belongs in the “tag” on the number line at the right? The tag is to the right of 0.75, so the fraction must be greater than 0.75. Express each fraction as a decimal.

\[
\begin{align*}
\frac{2}{3} &= 0.6, \\
\frac{3}{4} &= 0.75, \\
\frac{4}{5} &= 0.8, \\
\frac{9}{10} &= 0.9
\end{align*}
\]

Only 0.8 and 0.9 are greater than 0.75, and 0.9 is much closer to 1 than to 0.75. Choose 0.8, which is equal to \( \frac{4}{5} \).

On each number line, fill in the tags using the given fractions.

1. \( \frac{3}{8}, \frac{1}{2}, \frac{2}{3}, \frac{1}{9}, \frac{7}{8} \)

2. \( \frac{4}{3}, \frac{3}{4}, \frac{6}{5}, \frac{8}{15}, \frac{16}{15} \)

3. \( \frac{7}{4}, \frac{6}{5}, \frac{15}{8}, \frac{3}{2}, \frac{4}{3} \)

4. \( \frac{9}{5}, \frac{7}{3}, \frac{8}{5}, \frac{13}{6}, \frac{8}{4} \)

5. Write a fraction in simplest form for each tag on this number line. Use only the denominators 2, 3, 4, 5, 8, and 10. Express numbers greater than 1 as improper fractions.
Investigating Coordinate Grids

You can use coordinate grids to display sets of ordered pairs. You can also find new ordered pairs by looking at the line that the plotted ordered pairs make.

The table below lists the cost of tickets to a play. The data from the table are plotted on the grid.

<table>
<thead>
<tr>
<th>Number of Tickets</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$10.00</td>
</tr>
<tr>
<td>4</td>
<td>$20.00</td>
</tr>
<tr>
<td>6</td>
<td>$30.00</td>
</tr>
<tr>
<td>8</td>
<td>$40.00</td>
</tr>
</tbody>
</table>

The table shows the cost of 2, 4, 6, and 8 tickets. To find the cost of 5 tickets, you can use the grid to find the ordered pair that fits the table. Start at the origin and move to 5 on the x-axis. This is the x-coordinate. Move up until you meet the line. Then follow across to the left to the y-axis to find the corresponding y-coordinate. The value is 25. The ordered pair is (5, 25). This ordered pair means 5 tickets cost $25.

EXERCISES Use the data plotted on the coordinate grid to answer the questions.

<table>
<thead>
<tr>
<th>Time (in hours)</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>240</td>
</tr>
<tr>
<td>3</td>
<td>360</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
</tr>
<tr>
<td>8</td>
<td>960</td>
</tr>
</tbody>
</table>

1. How many miles did the airplane travel in 1 hour? _____
2. How many miles did the airplane travel in 2 hours? _____
3. How many miles did the airplane travel in 5 hours? _____
4. How long did it take the airplane to travel 720 miles? _____
**Greatest Possible Error**

When you measure a quantity, your measurement is more precise when you use a smaller unit of measure. But no measurement is ever exact—there is always some amount of error. The greatest possible error (GPE) of a measurement is one half the unit of measure.

**Example:**

- **unit of measure:** \(\frac{1}{8}\) inch
- **length of line segment:** \(1\frac{3}{8}\) inches
- **GPE:** half of \(\frac{1}{8}\) inch, or \(\frac{1}{16}\) inch

Since \(1\frac{3}{8} = 1\frac{6}{16}\), the actual measure of the line segment may range anywhere from \(1\frac{5}{16}\) inches to \(1\frac{7}{16}\) inches.

**Use the GPE to give a range for the measure of each line segment.**

1. 
2. 
3. 
4. 

5. Using this scale, the weight of a bag of potatoes is measured as 3 pounds. What is the range for the actual weight of the potatoes?

6. Using this container, the amount of a liquid is measured as 20 milliliters. What is the range for the actual amount of the liquid?
Using 1 as a Benchmark

When you estimate sums of proper fractions, it often helps to use the number 1 as a benchmark, like this:

Two halves make a whole, so \( \frac{1}{2} + \frac{1}{2} = 1 \).

If two fractions are each less than \( \frac{1}{2} \), their sum is less than 1.

If two fractions are each greater than \( \frac{1}{2} \), their sum is greater than 1.

\[
\frac{3}{8} + \frac{4}{9} < 1 \\
\frac{5}{8} + \frac{7}{9} > 1
\]

Fill in each \( < \) with \( > \) to make a true statement.

1. \( \frac{2}{3} + \frac{5}{8} \sqrt{1} 
2. \( \frac{2}{5} + \frac{3}{7} \sqrt{1} 
3. \( \frac{3}{10} + \frac{5}{11} \sqrt{1} 
4. \( \frac{27}{50} + \frac{7}{10} \sqrt{1} 
5. \( \frac{50}{99} + \frac{38}{75} \sqrt{1} 
6. \( \frac{24}{49} + \frac{32}{65} \sqrt{1}

Fill in each \( \sqrt{1} \) with one of the given fractions to make a true statement.

7. \( \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7} \)
8. \( \frac{8}{11}, \frac{7}{11}, \frac{6}{11}, \frac{5}{11} \)
9. \( \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \)
10. \( \frac{1}{25}, \frac{12}{25}, \frac{13}{25}, \frac{24}{25} \)

Fill in each with \( < \) or \( > \) to make a true statement.

11. \( \frac{5}{8} - \frac{1}{2} \sqrt{\frac{1}{2}} 
12. \( 1 - \frac{5}{11} \sqrt{\frac{1}{2}} 
13. \( 1 - \frac{10}{19} \sqrt{\frac{1}{2}} 
14. \( 1 - \frac{49}{99} \sqrt{\frac{1}{2}} 
15. \( 4 \frac{3}{7} + \frac{1}{3} \sqrt{5} 
16. \( 3 - \frac{4}{7} \sqrt{2 \frac{1}{2}} 

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In the puzzles below, the sum of the fractions in each row is the same as the sum of the fractions in each column. Use your knowledge of adding and subtracting fractions to find the missing fractions. 

**Hint:** Remember to check the fractions for like denominators before adding.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3/20</td>
<td>9/20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/20</td>
<td></td>
<td>2/20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/20</td>
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**CHALLENGE** Create your own fraction puzzle using a box of 5 rows and 5 columns.
Fractions are important in measurement. Using fractions allows you to be more exact than when you round to the nearest inch. When you go to the doctor, your height is not measured to the nearest inch. It is measured to fractions of an inch. Scientists use fractions all the time because their measurements need to be very precise.

**Solve.**

1. Janelle is cutting a piece of wood that is \( \frac{7}{12} \) inches long for a miniature picture frame. If she is cutting it from a piece of wood that is 1 inch long, what is the length of wood that will be left over?

   

2. The winning high jump in a track meet \( \frac{3}{8} \) inch away from the world record. The second place jump was \( \frac{7}{8} \) inch away from the world record. What is the difference between the two jumps?

   

3. A carpenter needs to fill a \( \frac{3}{4} \) inch-wide hole. He has a piece of wood that is \( \frac{9}{12} \) inch wide. How much should he cut off from the piece so that it will fit in the hole?

   

4. Evie is cutting ribbons \( 8\frac{1}{3} \) feet long for a sewing project. If the original ribbon is \( 36\frac{1}{4} \) feet long, how long is it after she cuts her first ribbon?

   

5. Fabric is sold by the yard. Derek wants \( \frac{3}{8} \) yard of a particular kind of fabric. There is only to be \( \frac{1}{4} \) yard of the fabric left on the bolt. Derek buys what is left. How much more does he need to buy?
Enrich

Unit Fractions

A **unit fraction** is a fraction with a numerator of 1 and a denominator that is any counting number greater than 1.

unit fractions: \( \frac{1}{2}, \frac{1}{3}, \frac{1}{10} \)

A curious fact about unit fractions is that each one can be expressed as a sum of two distinct unit fractions. (*Distinct* means that the two new fractions are different from one another.)

\[
\frac{1}{2} = \frac{1}{3} + \frac{1}{6} \quad \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \quad \frac{1}{10} = \frac{1}{11} + \frac{1}{110}
\]

1. The three sums shown above follow a pattern. What is it?

2. Use the pattern you described in Exercise 1. Express each unit fraction as a sum of two distinct unit fractions.
   
   a. \( \frac{1}{4} \)  
   b. \( \frac{1}{5} \)  
   c. \( \frac{1}{12} \)  
   d. \( \frac{1}{100} \)  

   Does it surprise you to know that other fractions, such as \( \frac{5}{6} \), can be expressed as sums of unit fractions? One way to do this is by using equivalent fractions. Here’s how.

   \[
   \frac{5}{6} = \frac{10}{12} \quad \rightarrow \quad \frac{10}{12} = \frac{6}{12} + \frac{4}{12} = \frac{1}{2} + \frac{1}{3} \quad \rightarrow \quad \frac{5}{6} = \frac{1}{2} + \frac{1}{3}
   \]

3. Express each fraction as a sum of two distinct unit fractions.

   a. \( \frac{2}{3} \)  
   b. \( \frac{4}{15} \)  
   c. \( \frac{5}{9} \)  
   d. \( \frac{2}{5} \)  

4. Express \( \frac{4}{5} \) as the sum of *three* distinct unit fractions.

5. **CHALLENGE** Show two different ways to express \( \frac{1}{2} \) as the sum of three distinct unit fractions.
Choose the Operation

Solve. Explain how you chose the operation.

1. A box is $\frac{1}{2}$ inches tall. If 5 of the boxes are stacked on top of each other, how tall is the stack of boxes?

2. Darlene needs $\frac{3}{4}$ yard of fabric to cover a chair. She has $\frac{3}{8}$ yard of fabric. How much more fabric does she need?

3. Mr. Montgomery is a chef. He has created 250 new recipes. He plans to donate $\frac{3}{5}$ of them to the school library. How many recipes does he plan to donate?

4. The art department received a shipment of 6 boxes of clay. Each box weighed $\frac{3}{4}$ pounds. How many pounds of clay were in the shipment?

5. A sculptor has a steel tube that is $\frac{2}{3}$ feet long. To create a longer tube, he attaches it to another steel tube that is $\frac{5}{6}$ feet long. How long is the new steel tube?

6. Marcel was in a triathlon, a race with 3 events. He ran 4 miles in $\frac{2}{3}$ hour. He bicycled 5 miles in $\frac{3}{4}$ hour, and he swam 880 yards in $\frac{1}{2}$ hour. What was his total race time?
Enrich

A Maze of Mixed Numbers

Find your way through the maze to reach the dot. When you come to a letter, solve the problem that matches that letter. Draw your path through the sum or difference in simplest form. Follow the path until you reach the next intersection. Continue finding each sum or difference.

A. \(3 \frac{1}{3} + 2 \frac{3}{5}\)  
B. \(\frac{1}{10} + 3 \frac{1}{2}\)  
C. \(4 \frac{1}{5} - 1 \frac{1}{6}\)  
D. \(4 \frac{1}{4} + 3 \frac{3}{4}\)

E. \(2 \frac{4}{5} + 5 \frac{1}{2}\)  
F. \(11 \frac{1}{3} - 1 \frac{1}{6}\)  
G. \(3 \frac{5}{6} + 2 \frac{5}{12}\)  
H. \(6 \frac{3}{5} + 2 \frac{1}{6}\)

I. \(7 \frac{1}{4} - 3 \frac{1}{5}\)  
J. \(8 \frac{3}{4} + 6 \frac{1}{3}\)  
K. \(1 \frac{2}{3} - 1 \frac{1}{3}\)  
L. \(8 \frac{7}{8} - 2 \frac{1}{4}\)

M. \(1 \frac{2}{3} + 9 \frac{3}{5}\)  
N. \(6 \frac{5}{8} - 1 \frac{1}{3}\)
Sometimes an equation involves both fractions and decimals. To solve an equation like this, you probably want to work with numbers in the same form. One method of doing this is to start by expressing the decimals as fractions. The example at the right shows how you might solve the equation \( m + \frac{2}{5} = 0.6 \).

Name the number that is a solution of the given equation.

1. \( z = \frac{1}{8} + 0.375; \frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4} \)  
2. \( 0.75 - \frac{3}{4} = b; 0, \frac{1}{4}, 1, \frac{1}{4} \)  
3. \( c + 0.6 = \frac{4}{5}, \frac{1}{5}, \frac{3}{5}, 1\frac{1}{5}, 1\frac{2}{5} \)  
4. \( 0.6 = j - \frac{1}{5}, \frac{1}{5}, 1, 1\frac{2}{5} \)  
5. \( \frac{1}{4} + r = 0.75; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \)  
6. \( d - 0.1 = \frac{7}{10}, \frac{1}{2}, \frac{3}{5}, \frac{4}{5}, \frac{9}{10} \)  

Solve each equation. If the solution is a fraction or a mixed number, be sure to express it in simplest form.

7. \( \frac{2}{5} + 0.4 = k \)  
8. \( s = \frac{7}{8} - 0.125 \)  
9. \( 0.6 - n = \frac{2}{5} \)  
10. \( t + 0.2 = \frac{4}{5} \)  
11. \( 0.375 + g = \frac{5}{8} \)  
12. \( y - 0.25 = \frac{3}{4} \)  
13. \( 0.8 - \frac{1}{5} = x \)  
14. \( q + 0.125 = \frac{5}{8} \)  
15. \( w = \frac{1}{8} + 0.375 + \frac{5}{8} \)  
16. \( 0.7 + \frac{1}{10} - 0.3 = a \)  
17. \( p + \frac{1}{5} = 0.8 - \frac{3}{5} \)  
18. \( k - 0.875 = 0.375 + \frac{1}{8} \)
Can you see a pattern in these multiplications?

\[
\begin{align*}
5.931 & \times 10 = 59.31 \\
5.931 & \times 100 = 593.1 \\
5.931 & \times 1,000 = 5,931
\end{align*}
\]

When you multiply a number by 10, 100, or 1,000, the product contains the same digits as the original number. However, the decimal point “moves” according to these rules.

- multiply by 10 \rightarrow move to the right one place
- multiply by 100 \rightarrow move to the right two places
- multiply by 1,000 \rightarrow move to the right three places

Many people use this fact as a mental math strategy.

**Find each product mentally.**

1. \(10 \times 7.402\) 
2. \(100 \times 7.402\) 
3. \(1,000 \times 7.402\) 
4. \(1,000 \times 0.5362\) 
5. \(100 \times 3.83\) 
6. \(24.07 \times 10\) 
7. \(1.918 \times 1,000\) 

**Estimate by rounding one number to 10, 100, or 1,000.**

9. \(6.57 \times 9\) 
10. \(1,225 \times 3.548\) 
11. \(0.6214 \times 11.05\) 
12. \(98.04 \times 26.331\) 

13. **CHALLENGE** Find the product \(1,000 \times 16.5\) mentally. How is this different from the other exercises on this page?
Here is a puzzle that will help you brush up on your logical thinking skills.

The product $3.3 \times 8.1$ is in both the circle and the triangle, but not in the square. Place the product in the diagram at the right.

\[
\begin{array}{c}
8.1 \\
\times 3.3 \\
\hline \\
243 \\
\hline \\
26.73
\end{array}
\]

Write 26.73 in the correct region of the diagram.

Use the given information to place the product in the diagram above.

1. The product $14.19 \times 1.3$ is in both the triangle and the square, but not in the circle.
2. The product $0.08 \times 2.7$ is in the triangle, but not in the circle or the square.
3. The product $1.24 \times 0.16$ is not in the circle, the square, or the triangle.
4. The product $2.2 \times 0.815$ is in both the square and the circle, but not in the triangle.
5. The product $0.02 \times 0.03$ is in the circle, but not the triangle or the square.
6. The product $21.7 \times 0.95$ is in the circle, the square, and the triangle.
7. The product $2.5 \times 12.8$ is in the square, but not the circle or triangle.
8. If you did all the calculations correctly, the sum of all the numbers in the diagram should be a “nice” number. What is the sum?
**Enrich**

**Better Buy**

Play this game with a partner. Take turns. You will need a number cube and counters.

**How to Play**
- Place your counter on Start. Roll the number cube and move the number of spaces rolled.
- Determine which item is the better buy. Have your partner check your answer.
- If you are wrong, you must go back 2 spaces.

The first person to get to the Finish square is the winner.

<table>
<thead>
<tr>
<th></th>
<th>Bread</th>
<th>Cereal</th>
<th>Free Space</th>
<th>Canned Vegetables</th>
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<tbody>
<tr>
<td>Start</td>
<td>16 oz for $0.89 or 20 oz for $1.29</td>
<td>13 oz for $2.29 or 20 oz for $3.29</td>
<td></td>
<td>5 oz for $3 or 3 for $2</td>
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<tr>
<td></td>
<td>Potatoes</td>
<td>Bananas</td>
<td></td>
<td>Tuna</td>
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<tr>
<td></td>
<td>5 lb for $2 or 3 lb for $1.50</td>
<td>3 lb for $1 or 2 lb for $0.69</td>
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<td>6 oz can for $1.29 or 12 oz can for $2.39</td>
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<td>Salad Dressing</td>
<td>Donuts</td>
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<td>Crackers</td>
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<td></td>
<td>8 oz for $0.99 or 16 oz for $1.80</td>
<td>12 for $4.79 or 6 for $2.59</td>
<td></td>
<td>12 oz for $1.79 or 16 oz for $2.49</td>
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<td>Rolls</td>
<td>Detergent</td>
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<td>Yogurt</td>
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<td></td>
<td>12 for $1.99 or 18 for $2.69</td>
<td>100 oz for $4.98 or 50 oz for $2.29</td>
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<td>8 oz for $0.79 or 6 oz for $0.49</td>
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<td></td>
<td>Orange Juice</td>
<td>Peanut Butter</td>
<td></td>
<td>Mustard</td>
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<td></td>
<td>64 oz for $2.59 or 32 oz for $1.49</td>
<td>24 oz for $2.99 or 18 oz for $1.99</td>
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<td>40 oz for $2.39 or 28 oz for $1.79</td>
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<td></td>
<td>Soap</td>
<td>Noodles</td>
<td></td>
<td>Oil</td>
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<tr>
<td></td>
<td>8 bars for $5 or 3 bars for $1.49</td>
<td>12 oz for $0.99 or 16 oz for $1.39</td>
<td></td>
<td>48 oz for $1.99 or 32 oz for $1.39</td>
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*FREE SPACE*
**Unit Pricing**

The unit price of an item is the cost of the item given in terms of one unit of the item. The unit might be something that you count, like jars or cans, or it might be a unit of measure, like ounces or pounds. You can find a unit price using this formula.

\[
\text{unit price} = \frac{\text{cost of item}}{\text{number of units}}
\]

For example, you find the unit price of the tuna in the ad at the right by finding the quotient 0.89 ÷ 6. The work is shown below the ad. Rounding the quotient to the nearest cent, the unit price is $0.15 per ounce.

### Find a unit price for each item.

- **1.** 5-pound bag CARROTS
  - 5-pound bag
  - CARROTS
  - $1.29

- **2.** 18-ouncer jar PEANUT BUTTER
  - 18-ouncer jar
  - PEANUT BUTTER
  - $2.49

- **3.** Grade A Jumbo EGGS
  - Grade A Jumbo
  - EGGS
  - Dozen $1.59

### Give two different unit prices for each item.

- **4.** Frozen BURRITOS
  - 5-ounce pkg
  - 2 for $1.39

- **5.** Purr-fect CAT FOOD
  - 3/1 3-ounce can

- **6.** Old Tyme SPAGHETTI SAUCE
  - 12-ounce jars
  - 2/$3

### Circle the better buy.

- **7.** Mozarella Cheese
  - 3/$4
  - 10-ounce pkg

- **8.** Dee-light Chicken Wings
  - $9.99
  - 5-pound bag

- **9.** Top Q Chicken Wings
  - $2.29
  - 18-ounce bag
Enrich

It’s in the Cards

Below each set of cards, a quotient is given. Use the digits on the cards to form a division sentence with that quotient. Use as many zeros as you need to get the correct number of decimal places. For example, this is how to find a division sentence for the cards at the right.

You know that \( 24 \div 3 = 8 \).
So, one division is \( 0.0024 \div 30 = 0.0008 \).

1. Quotient: 0.009
   
   2. Quotient: 0.04
   
   3. Quotient: 0.0005
   
   4. Quotient: 0.0074
   
   5. Quotient: 0.0155
   
   6. Quotient: 0.0025
   
   7. Quotient: 0.0004
   
   8. Quotient: 0.03
   
   9. Quotient: 0.005
   
   10. Quotient: 20.65
   
   11. Quotient: 0.0208
   
   12. Quotient: 0.08

13. **CHALLENGE** Use the cards at the right. Write four different divisions that have the quotient 0.4.
Suppose that you are meeting a friend for lunch and come across the sale advertised at the right. For weeks, you have wanted to buy a set of CDs that is regularly priced at $31.98. Here is how compatible numbers can help you find the sale price of the set.

- \( \frac{1}{4} \) of $31.98 is about \( \frac{1}{4} \) of $32, or $8.
- \( \frac{1}{4} \text{ off} \) means that you pay \( 1 - \frac{1}{4} \) or \( \frac{3}{4} \).
- Since \( \frac{1}{4} \text{ of }$32 = $8, \( \frac{3}{4} \text{ of }$32 = $24 \).

The sale price is about $24.

Each exercise gives the regular price of one or more items. Use the information at the right to estimate the sale price.

1. video game: $23.95
2. CD: $15.95
3. headphones: $10.98
4. three packs of TRUE-CELL batteries; $5.98 per pack
5. one CD: $20.95
   one video game: $27.99
6. one set of headphones: $15.79
   two video games: $17.55 and $15.50
7. one CD: $16.95
   one set of headphones: $14.50
   one DVD: $19.98
Enrich

Mixed Numbers and Mental Math

Sometimes you can multiply a whole number and a mixed number in your head. Think of the mixed number in two parts—the whole number and the fraction.

Find each product mentally.

**Example**

Think: \(3 \times 10\)  
Think: \(\frac{1}{2}\) of 10

\[
\frac{3}{2} \times 10 = \quad + \quad =
\]

1. \(\frac{7}{2} \times 6 = \quad + \quad = \)
2. \(4 \times 9\frac{1}{2} = \quad + \quad = \)
3. \(4\frac{1}{3} \times 6 = \quad + \quad = \)
4. \(5\frac{1}{4} \times 8 = \quad + \quad = \)
5. \(15 \times 2\frac{1}{5} = \quad + \quad = \)

Now you can use this mental math technique to make better estimates. Here’s how.

Estimate the product: \(4\frac{1}{2} \times 11\frac{7}{9}\)

\[
4\frac{1}{2} \times 11\frac{7}{9} \rightarrow 4\frac{1}{2} \times 12
\]

So, \(4\frac{1}{2} \times 11\frac{7}{9}\) is about 54.

\[
4\frac{1}{2} \times 12 = 4 \times 12 + \frac{1}{2} \text{ of } 12
\]

\[
= 48 + 6
\]

\[
= 54
\]

Estimate each product.

6. \(6\frac{1}{2} \times 4\frac{2}{11}\)
7. \(5\frac{1}{3} \times 8\frac{9}{10}\)
8. \(11\frac{15}{16} \times 2\frac{1}{4}\)
9. \(5\frac{7}{10} \times 4\frac{1}{6}\)
Sometimes an operation involves both fractions and decimals. To perform the operation, you need to express all the numbers in the same form. Here are two examples.

\[
\frac{1}{5} \div 0.3 = \frac{1}{5} \div \frac{1}{3} \quad \leftarrow \text{Express the decimal as a fraction}
\]

\[
= \frac{1}{5} \times \frac{3}{1}
\]

\[
= \frac{3}{5}
\]

\[
\frac{3}{4} + 0.115 = 0.75 + 0.115 \quad \leftarrow \text{Express the fraction as a decimal}
\]

\[
= 0.865
\]

**Perform the operation. Write in simplest form.**

1. \( \frac{5}{16} \div 0.25 \)
2. \( 0.6 \div \frac{7}{9} \)
3. \( 0.125 \times \frac{4}{11} \)
4. \( 1 \frac{1}{5} \times 0.3 \)
5. \( 0.8 - \frac{3}{5} \)
6. \( 1 \frac{3}{8} - 0.875 \)

**Perform the operation. Express the answer as a decimal.**

7. \( 0.34 \div \frac{1}{5} \)
8. \( \frac{1}{8} \div 0.005 \)
9. \( 0.001 \times \frac{3}{5} \)
10. \( 6.39 + \frac{7}{8} \)
11. \( 9.1 - \frac{1}{4} \)
12. \( \frac{3}{8} + 0.709 + \frac{2}{5} \)

13. Kevin is making one recipe that calls for \( 1 \frac{1}{4} \) pounds of hamburger and another that calls for 2 pounds. In the store, he finds a family pack of hamburger that is labeled 3.75 pounds. Is this more or less than he needs? How much more or less?

14. Daneesha needs \( 1 \frac{1}{2} \) yards of material to make a jacket and \( 1 \frac{3}{4} \) yards of material to make a skirt. The material costs $7.50 per yard. What is the total cost of the material for the skirt and jacket? Round your answer to the nearest cent.
Rewrite the recipe for a new serving size.

**Pancakes (serves 6)**

3\(\frac{3}{4}\) cups flour

1\(\frac{1}{2}\) tsp. salt

4 eggs

1\(\frac{3}{4}\) cup milk

1\(\frac{3}{4}\) T baking powder

\(\frac{1}{2}\) cup sugar

1\(\frac{1}{2}\) T canola oil

**Pancakes (serves 8)**

______ cups flour

______ T baking powder

______ tsp. salt

______ cup sugar

______ eggs

______ T canola oil

______ cup milk
Enrich

Modeling Division of Fractions on a Ruler

How many half-inch lengths are in 4 inches? When you look at a ruler, it is easy to see that the answer is 8.

\[ \frac{4}{\frac{1}{2}} = 8 \]

So, this diagram is also a model for the division, \( 4 \div \frac{1}{2} = 8 \).

Write the division that is modeled in each diagram.

1. 
   \[ \frac{\text{INCHES}}{1 \ 2 \ 3 \ 4 \ 5} \]
   
   

2. 
   \[ \frac{\text{INCHES}}{1 \ 2 \ 3 \ 4 \ 5} \]
   
   

3. 
   \[ \frac{\text{INCHES}}{1 \ 2 \ 3 \ 4 \ 5} \]
   
   

4. 
   \[ \frac{\text{INCHES}}{1 \ 2 \ 3 \ 4 \ 5} \]
   
   

5. Use the ruler below. Create a model for the division \( 4 \frac{2}{3} \div \frac{2}{3} = 7 \).
Riddle: What do the emu, the cassowary, and the ostrich have in common?

To find out, find the following quotients. Then, find the quotients at the bottom of the page and put the letter of each above the answer.

**F.** $\frac{8}{2} \div \frac{3}{3} = \underline{\phantom{0000}}$

**Y.** $\frac{3}{4} \div \frac{3}{5} = \underline{\phantom{0000}}$

**D.** $\frac{5}{8} \div 9 = \underline{\phantom{0000}}$

**A.** $\frac{7}{12} \div \frac{5}{9} = \underline{\phantom{0000}}$

**E.** $\frac{5}{9} \div \frac{15}{5} = \underline{\phantom{0000}}$

**L.** $\frac{3}{16} \div \frac{7}{8} = \underline{\phantom{0000}}$

**I.** $26 \div \frac{7}{9} = \underline{\phantom{0000}}$

**B.** $4\frac{1}{2} \div 6\frac{2}{3} = \underline{\phantom{0000}}$

**H.** $35 \div \frac{5}{7} = \underline{\phantom{0000}}$

**C.** $4\frac{2}{3} \div 18 = \underline{\phantom{0000}}$

**T.** $8\frac{1}{6} \div 2\frac{1}{3} = \underline{\phantom{0000}}$

**R.** $9 \div \frac{4}{9} = \underline{\phantom{0000}}$

**N.** $2\frac{2}{9} \div \frac{4}{5} = \underline{\phantom{0000}}$

**S.** $\frac{7}{12} \div 42 = \underline{\phantom{0000}}$
1. Write an integer to show the location for each item.

- porpoise ______
- sea horse ______
- bird ______
- octopus ______
- eel ______
- clouds ______
- flag ______
- jellyfish ______

2. Write the opposite for each integer. Tell what can be found in that location.

- 2 ______
- 7 ______
- -4 ______
- 4 ______
- 9 ______
- -6 ______
- -3 ______
- 6 ______

3. Compare the locations of each pair. Circle the location showing the greater integer.

- sea horse, porpoise
- clouds, eel
- eel, flag
- sail of boat, bottom of ocean
- buoy, octopus
- bird, sea horse

4. Order the integers from least to greatest.

- -9, 6, 3, -2, -12, 7, -8, 1, 10, 0, -11, -3
Enrich

Adding Integers

One African-American inventor of the late nineteenth century had many patents. Who was he and which patent of 1890 became a common household item?

Find the sum of the number in the center of each ring and each of the numbers in the middle ring. Write the sum in the corresponding space in the outer ring.

1. 

2. 

3. 

4. 

5. 

6. 

Circle the letter of each negative sum in the outer ring. Starting at the arrow and moving in a clockwise direction, write each circled letter in the blanks below.

The inventor’s name was __________ __________ __________ __________ __________ __________

and his most famous invention was the __________ __________ __________ __________ __________ __________
I am a two-digit number. My tens digit is greater than my ones digit. The difference of my digits is 1.

Shade in the boxes that contain a correct answer below the subtraction. Now you can find out what number I am.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(−18)</td>
<td>28</td>
<td>(−13)</td>
<td>68</td>
<td>8</td>
<td>(−69)</td>
</tr>
<tr>
<td>−14</td>
<td>−(−21)</td>
<td>−(−25)</td>
<td>−69</td>
<td>−(−20)</td>
<td>−11</td>
</tr>
<tr>
<td>−32</td>
<td>7</td>
<td>12</td>
<td>1</td>
<td>28</td>
<td>−80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>(−8)</td>
<td>48</td>
<td>12</td>
<td>(−20)</td>
<td>(−2)</td>
</tr>
<tr>
<td>−65</td>
<td>−48</td>
<td>−66</td>
<td>−13</td>
<td>−(−13)</td>
<td>−12</td>
</tr>
<tr>
<td>−12</td>
<td>−(−40)</td>
<td>−18</td>
<td>−1</td>
<td>−(−42)</td>
<td>−14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>(−40)</td>
<td>16</td>
<td>(−30)</td>
<td>45</td>
<td>(−2)</td>
</tr>
<tr>
<td>−(−21)</td>
<td>−50</td>
<td>−2</td>
<td>−42</td>
<td>−65</td>
<td>−(−12)</td>
</tr>
<tr>
<td>41</td>
<td>10</td>
<td>18</td>
<td>−12</td>
<td>−20</td>
<td>−14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(−33)</td>
<td>13</td>
<td>(−52)</td>
<td>(−86)</td>
<td>7</td>
<td>42</td>
</tr>
<tr>
<td>−(−25)</td>
<td>−15</td>
<td>−8</td>
<td>−(−90)</td>
<td>−(−27)</td>
<td>−(−53)</td>
</tr>
<tr>
<td>−(−58)</td>
<td>2</td>
<td>−(−60)</td>
<td>−(−4)</td>
<td>−(−20)</td>
<td>−(−11)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(−15)</td>
<td>41</td>
<td>(−3)</td>
<td>(−18)</td>
<td>29</td>
<td>(−45)</td>
</tr>
<tr>
<td>−18</td>
<td>−(−11)</td>
<td>−(−18)</td>
<td>−(−13)</td>
<td>−(−15)</td>
<td>−11</td>
</tr>
<tr>
<td>−3</td>
<td>15</td>
<td>5</td>
<td>5</td>
<td>(−4)</td>
<td>(−56)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−22)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−26)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−37)</td>
<td></td>
</tr>
</tbody>
</table>
3. **Negative Exponents**

Magic square entries can be converted to exponents to create Exponent Squares, which can be made into Multi-Magic Squares. The products of the numbers in each row, column, or diagonal are the same in a Multi-Magic Square.

**Addition Square**

<table>
<thead>
<tr>
<th>6</th>
<th>2</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

**Convert to exponents.**

<table>
<thead>
<tr>
<th>2^{-1}</th>
<th>2^{4}</th>
<th>2^{3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2^{6}</td>
<td>2^{2}</td>
<td>2^{-2}</td>
</tr>
<tr>
<td>2^{1}</td>
<td>2^{0}</td>
<td>2^{5}</td>
</tr>
</tbody>
</table>

**Complete Multi-Magic Square.**

<table>
<thead>
<tr>
<th>1/2</th>
<th>16</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>4</td>
<td>1/4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>32</td>
</tr>
</tbody>
</table>

Find the product of the Multi-Magic Square above. _____

Complete the squares.

1. **Addition Square**

<table>
<thead>
<tr>
<th>-2</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>4</td>
</tr>
</tbody>
</table>

**Convert to exponents.**

<table>
<thead>
<tr>
<th>3^{0}</th>
<th>3^{5}</th>
<th>3^{-2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>3^{-1}</td>
<td>3^{1}</td>
<td>3^{3}</td>
</tr>
<tr>
<td>3^{4}</td>
<td>3^{-3}</td>
<td>3^{2}</td>
</tr>
</tbody>
</table>

**Multi-Magic Square**

Product: _____

2. **Addition Square**

<table>
<thead>
<tr>
<th>3^{-1}</th>
<th>3^{1}</th>
<th>3^{3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>3^{4}</td>
<td>3^{-3}</td>
<td>3^{2}</td>
</tr>
</tbody>
</table>

Product: _____
If the solutions to the equations are incorrect, cross out the letters in the box below. The remaining letters will spell out a message.

| B. | 2x + 1 + 4 = 9 | P. | 7 = \( \frac{2}{9} c - 1 \) | c = 18 |
| I. | 3m + 4m = 21 | C. | 9 = \( \frac{x}{14} \) - 23 | x = 184 |
| L. | 13 = 5 + 4y | O. | 2(y - 7) = 0 | y = 7 |
| X. | 2(x + 1) = 5 | K. | 6(a + 4) = 110 | a = 16 |
| W. | 2 + 3x = 5 | F. | \( \frac{3}{4} m + 1 = 9 \) | m = 12 |
| S. | 8x - 3x = 20 | E. | 5n - 21 = 24 | n = 9 |
| M. | 15 = 2(n + 1) | H. | 2(y - 6) = 4 | y = 8 |
| N. | 11 = 3 + 4y | J. | 2x - 8 = 10 | x = 8 |
| R. | 4m + 2m + 3 = 15 | Q. | 3(x - 7) = 32 | x = 13 |
| G. | \( \frac{3}{7} n - 5 = 4 \) | T. | 6(m - 35) = 6 | m = 36 |
| D. | 27 = 11p + 9 | U. | 78 = 9w - 7w + 6 | w = 36 |
| V. | \( \frac{5}{8} a + 30 = 35 \) | Y. | \( \frac{3}{5} f - 7 = 8 \) | f = 25 |

<table>
<thead>
<tr>
<th>P</th>
<th>Y</th>
<th>L</th>
<th>B</th>
<th>O</th>
<th>M</th>
<th>Q</th>
<th>U</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>F</td>
<td>C</td>
<td>A</td>
<td>K</td>
<td>V</td>
<td>J</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>K</td>
<td>T</td>
<td>H</td>
<td>L</td>
<td>E</td>
<td>P</td>
<td>R</td>
<td>I</td>
<td>F</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
<td>M</td>
<td>H</td>
<td>D</td>
<td>T</td>
<td>X</td>
<td>L</td>
<td>A</td>
</tr>
<tr>
<td>B</td>
<td>N</td>
<td>D</td>
<td>M</td>
<td>S</td>
<td>P</td>
<td>W</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>J</td>
<td>B</td>
<td>E</td>
<td>L</td>
<td>M</td>
<td>D</td>
<td>F</td>
<td>R</td>
<td>K</td>
</tr>
</tbody>
</table>
Enrich

Multiply and Divide Integers

Evaluate each numerical expression. Put the letters above the answers at the bottom of the page to answer the questions.

Y. \(-52 \div 4 = \underline{\quad}\)  
A. \(-72 \div (-8) = \underline{\quad}\)

S. \(-24 \div (-12) = \underline{\quad}\)  
L. \(-100 \div (-20) = \underline{\quad}\)

N. \((-3 + -9) \div 4 = \underline{\quad}\)  
V. \(6 + (-15) \div 5 = \underline{\quad}\)

R. \((8 \times -8) \div (4 \times -4) = \underline{\quad}\)  
L. \(30 \div (-5) + 40 \div (-8) = \underline{\quad}\)

K. \((21 - -7) \div 4 = \underline{\quad}\)  
I. \((-6 \times -8) \div (2 - 6) = \underline{\quad}\)

T. \(15 - (-25) \div 5 = \underline{\quad}\)  
U. \(-45 \div 5 + 24 \div (-3) = \underline{\quad}\)

A. \(-49 \div 7 - (-3) = \underline{\quad}\)  
H. \((12 \times -6) \div (-4 - 4) = \underline{\quad}\)

N. \((2 + 50) \div (-2) = \underline{\quad}\)

Who was the first woman to walk in space?

\[
\begin{array}{cccccccc}
7 & -4 & 20 & 9 & 4 & -13 & -3 \\
-17 & 5 & -11 & -12 & 3 & 9 & -26 \\
\end{array}
\]
A magic square is a square of numbers in which each row, column, and diagonal has the same sum.

Use counters to find if each is a magic square. If so, give the magic sum.

1. \[\begin{array}{ccc}
-4 & 10 & 0 \\
-6 & 2 & -2 \\
4 & 6 & -8 \\
\end{array}\]

Magic Square: Yes or No? ____

2. \[\begin{array}{ccc}
-7 & -12 & -11 \\
-14 & -10 & -6 \\
-9 & -8 & -13 \\
\end{array}\]

Magic Square: Yes or No? ____

Use counters to complete these magic squares.

3. \[\begin{array}{ccc}
7 & -7 & 4 \\
2 & 1 & -1 \\
-2 & -3 & \\
-5 & 6 & -8 \\
\end{array}\]

Sum: ____

Sum of the four middle squares: ____

Sum of the four corners: ____

4. \[\begin{array}{ccc}
-7 & -8 & 3 \\
-5 & 0 & 1 \\
-1 & -3 & 2 \\
-6 & 5 & 4 & -9 \\
\end{array}\]

Sum: ____

Sum of the four middle squares: ____

Sum of the four corners: ____
Latitude lines are parallel to the equator and run from 90°N (the North Pole) to 90°S (the South Pole). Longitude lines are measured east and west of the prime meridian, the 0° longitude line which runs through Greenwich, England. Point A in the figure has coordinates (15°S, 45°W).

Write the coordinates, giving the north-south coordinates first.

1. B ____________
2. C ____________
3. D ____________
4. E ____________
5. F ____________
6. South Pole ____________

Graph each point on the coordinate grid map. Write the letter beside the point.

7. G (75°S, 75°E) 8. H (25°N, 45°E)
9. I (60°S, 90°W) 10. J (0°, 30°E)
11. K (60°N, 75°E) 12. L (45°N, 60°W)

13. How many degrees of latitude separate Halifax, Nova Scotia (45°N, 65°W), and Cordoba, Argentina (32°S, 65°W)? ____________

14. How many degrees of longitude separate Baku (41°N, 50°E), and New Haven, CT (41°N, 73°W)? ____________

15. One degree of longitude at the equator is about 69.2 miles. How far is it from Quito, Equador (0°, 79°W) to Kampala, Uganda (0°, 32°11/2°E), to the nearest 10 miles? ____________

16. Macapa, Brazil (0°, 51°W), is located almost exactly due west of Libreville, Gabon. The distance between the cities is about 4,150 miles. Give the latitude and longitude of Libreville, to the nearest degree. ____________
Explore Addition Equations

Solve each equation. Then write each letter above its number at the bottom of the page.

1. \(15 + B = 23\) ______
2. \(S + 9 = 18\) ______
3. \(H + 8 = 24\) ______
4. \(S + 3 = 13\) ______
5. \(L + 6 = 18\) ______
6. \(E + 7 = 13\) ______
7. \(P + 9 = 27\) ______
8. \(T + 17 = 32\) ______
9. \(S + 26 = 40\) ______
10. \(N + 57 = 128\) ______
11. \(16 + O = 27\) ______
12. \(E + 516 = 741\) ______
13. \(H + 13 = 20\) ______
14. \(I + 424 = 623\) ______
15. \(15 + O = 34\) ______
16. \(39 + Y = 73\) ______
17. \(E + 249 = 355\) ______
18. \(38 + T = 91\) ______
19. \(T + 55 = 72\) ______
20. \(49 + Y = 106\) ______
21. \(I + 804 = 976\) ______
22. \(125 = C + 112\) ______

What words of advice could you give to Pinocchio?
Enrich

Addition and Subtraction Equations

Play this game with a partner. You will need counters, scissors, and a paper bag.

• Cut out the equations at the bottom of the page and put them in the paper bag.

• Draw an equation from the bag. Calculate the answer. If the answer is a whole number, move forward that many spaces. If the answer is a fraction or decimal, go back to START.

• The winner is the first player to reach 50 points. You may need to go around the game board several times before there is a winner.

Subtract 10 from your score. | Go ahead 3 spaces. | Lose a turn. | Add your partner’s score to yours. | You earn 9 points.
--- | --- | --- | --- | ---
Add 6 to everyone’s score. | You earn 7 points. |
You earn 7 points. |
Take another turn. |
Go back 2 spaces. | You lose 2 points. | You earn 8 points. | Start |

The Equation Game

- $x + 19 = 24$
- $a + 14 = 18$
- $m + 7.5 = 8.5$
- $p - 2 = 5$
- $f - 2 = 1$
- $y - 5 = 1$
- $n - 1 = 3$
- $w - 2.8 = 0.2$
- $d + 3\frac{5}{6} = 5\frac{5}{6}$
- $h + 4.6 = 6.4$
- $b + \frac{1}{3} = 4\frac{1}{3}$
- $c + 5.3 = 7.3$
- $w - 2.8 = 0.2$
- $d - 0 = 5\frac{3}{4}$
- $q - 2.9 = 1.1$
- $j + 1.8 = 2.2$
- $v + 1 = 7$
- $r - 5 = 0$
- $t - 1.3 = 0.7$
The Vice President takes over if the President cannot fulfill the term of office. But who takes over should the Vice President leave office before completing the term of office?

To find the answer, solve each equation. Write the capital letter that is before the equation on the line above the answer at the bottom of the page.

<table>
<thead>
<tr>
<th>H</th>
<th>$8n = 40$</th>
<th>$n = \underline{\hspace{2cm}}$</th>
<th>S</th>
<th>$3c = 36$</th>
<th>$c = \underline{\hspace{2cm}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>$\frac{y}{3} = 9$</td>
<td>$y = \underline{\hspace{2cm}}$</td>
<td>P</td>
<td>$10k = 140$</td>
<td>$k = \underline{\hspace{2cm}}$</td>
</tr>
<tr>
<td>O</td>
<td>$3d = 12.9$</td>
<td>$d = \underline{\hspace{2cm}}$</td>
<td>U</td>
<td>$\frac{y}{8} = 9$</td>
<td>$y = \underline{\hspace{2cm}}$</td>
</tr>
<tr>
<td>V</td>
<td>$9r = 27$</td>
<td>$r = \underline{\hspace{2cm}}$</td>
<td>N</td>
<td>$2.3t = 0.46$</td>
<td>$t = \underline{\hspace{2cm}}$</td>
</tr>
<tr>
<td>T</td>
<td>$4t = 38$</td>
<td>$t = \underline{\hspace{2cm}}$</td>
<td>I</td>
<td>$87t = 87$</td>
<td>$t = \underline{\hspace{2cm}}$</td>
</tr>
<tr>
<td>E</td>
<td>$0.6n = 36$</td>
<td>$n = \underline{\hspace{2cm}}$</td>
<td>F</td>
<td>$\frac{a}{5} = 11$</td>
<td>$a = \underline{\hspace{2cm}}$</td>
</tr>
<tr>
<td>K</td>
<td>$\frac{11}{3}a = 12$</td>
<td>$a = \underline{\hspace{2cm}}$</td>
<td>A</td>
<td>$13d = 16.9$</td>
<td>$d = \underline{\hspace{2cm}}$</td>
</tr>
</tbody>
</table>
Ratios and Rectangles

1. Use a centimeter ruler to measure the width and the length of each rectangle. Then express the ratio of the width to the length as a fraction in simplest form.

   A

   B

   C

2. Similar figures have the same shape, but not necessarily the same size. Two rectangles are similar if the ratio of the width to the length is the same for each. Which rectangles in Exercise 1 are similar? __________

3. For centuries artists and architects have used a shape called the golden rectangle because people seem to find it most pleasant to look at. In a golden rectangle, the ratio of the width to the length is a little less than 5/8. Which rectangle in Exercise 1 is most nearly a golden rectangle? __________
Solve.

1. Diane runs 5 miles in 45 minutes. Claire runs 7 miles in 56 minutes. Who runs at a faster rate? Explain.

2. Lance rides 84 miles in 6 hours. Nathan rides 104 miles in 8 hours. Who rides at a faster rate? Explain.

3. Annie hikes 51 miles in 3 days. Alex hikes 64 miles in 4 days. Who hikes at a faster pace? Explain.

4. Jake’s car uses 5 gallons of gas to drive 115 miles in the city. The car uses 6 gallons of gas to drive 174 miles on the highway. Does Jake’s car get better gas mileage in the city or on the highway? Explain.

5. Oscar’s car uses 8 gallons of gas to drive 256 miles. Randy’s car uses 7 gallons of gas to drive 252 miles. Whose car gets better gas mileage? Explain.

6. Last week Vicky swam 16 laps in 20 minutes. This week she swam 20 laps in 16 minutes. Which week did she swim faster? Explain.
Enrich

The Brownie Business

Julie has decided that she wants to start a brownie business to make extra money over the summer. Before she can ask her parents for money to start her business, she needs to have some information about how many batches of brownies she can make in a day and for how much she must sell the brownies to make a profit.

1. Julie can bake 3 batches of brownies in 2 hours. Her goal is to bake 12 batches of brownies each day. Use the table to find how many hours Julie will need to bake to reach her goal.

<table>
<thead>
<tr>
<th>Batches of Brownies</th>
<th>3</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2. Each batch of brownies will be sold for $2. How much money will Julie make if she sells 6 batches of brownies?

<table>
<thead>
<tr>
<th>Batches of Brownies</th>
<th>1</th>
<th></th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. If Julie works for 10 hours a day, how many batches of brownies can she bake?

<table>
<thead>
<tr>
<th>Batches of Brownies</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>2</td>
</tr>
</tbody>
</table>

4. If Julie hires a friend, they can bake 24 batches of brownies in 8 hours. If they both work 40 hours in one week, how many batches of brownies can they bake that week? If Julie still charges $2 a batch, how much money will they make that week?

<table>
<thead>
<tr>
<th>Hours</th>
<th>8</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batches of Brownies</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Batches of Brownies</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2</td>
</tr>
</tbody>
</table>
“Liberty Enlightening the World”

On July 4, 1889, in gratitude to the French for the gift of the Statue of Liberty, Americans from Paris gave to the French a miniature Statue of Liberty. The statue is made of bronze and is approximately one fourth the size of the original. This smaller-scale copy is found near the Grenelle Bridge on the Île des Cygnes, an island in the Seine River about one mile south of the Eiffel Tower.

1. If the original Statue of Liberty is approximately 150 feet tall, about how tall is the replica?

2. Complete the table. The first one is done for you.

<table>
<thead>
<tr>
<th>Original Statue of Liberty</th>
<th>Replica</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of hand</td>
<td>16 ft</td>
</tr>
<tr>
<td>Length of nose</td>
<td>4.5 ft</td>
</tr>
<tr>
<td>Length of right arm</td>
<td>42 ft</td>
</tr>
<tr>
<td>Head thickness from ear to ear</td>
<td>2.5 ft</td>
</tr>
<tr>
<td>Width of mouth</td>
<td></td>
</tr>
<tr>
<td>Thickness of waist</td>
<td>35 ft</td>
</tr>
<tr>
<td>Distance from heel to the top of her head</td>
<td>111 ft</td>
</tr>
<tr>
<td>Length of index finger</td>
<td>8 ft</td>
</tr>
<tr>
<td>Circumference of the second joint</td>
<td>3.5 ft</td>
</tr>
</tbody>
</table>

3. The fingernail on the index finger of the original weighs 1.5 kilograms. How much does the fingernail on the replica in France weigh?

4. The dimensions of the tablet that Lady Liberty is holding are 23.6 feet by 13.6 feet by 2 feet. What are the dimensions of the smaller-scale tablet in France?

5. **Challenge** The fingernail on the index finger is 13 inches long and 10 inches wide. What will be the area of the fingernail on the replica in France?
Choose the best strategy to solve the problem. Tell what strategy you used.

1. Helena was making a beaded necklace. She strung one blue bead, three red beads, and two green beads. Then she strung two blue beads, four red beads, and three green beads. What combination of beads will she string next?

2. The Hernandez family is posing for a picture. The family has a mother, father, two children, and a grandmother. They will sit on the sofa in their living room for the picture. How many different ways can the Hernandez family sit on the sofa for the shot?

3. James is studying for the spelling bee championship. The championship takes place in 15 days. The first day, he memorizes 5 words. Each day after that, he memorizes 5 more words. How many words in all will James know on the day before the championship?

4. Kelly swam 15 laps in the swimming pool. It took her $7\frac{1}{2}$ minutes. If her pool is 36 feet long, how long would it take her to swim across a lake that is 48 yards wide?

5. An elevator can safely carry 500 pounds. On the first floor, two 125-pound women got on the elevator. On the second floor, one of the women got off, and a 215-pound man got on. On the third floor, a 140-pound woman got on with her 12-pound baby. On the fourth floor, the 125-pound woman got off, but another woman wanted to get on. What is the most the woman can weigh for her to safely board the elevator?
Did you know that a woman wrote the first description of a computer programming language? She was the daughter of a famous English lord and was born in 1815. She had a deep understanding of mathematics and was fascinated by calculating machines. Her interests led her to create the first algorithm. In 1843, she translated a French version of a lecture by Charles Babbage. In her notes to the translation, she outlined the fundamental concepts of computer programming. She died in 1852. In 1979, the U.S. Department of Defense named the computer language *Ada* after her.

To find out this woman’s full name, solve the equation for each letter.

1. \( \frac{7}{A} = \frac{28}{40} \)
2. \( \frac{5}{4} = \frac{B}{36} \)
3. \( \frac{1}{3} = \frac{C}{15} \)
4. \( \frac{5}{D} = \frac{35}{63} \)
5. \( \frac{2}{5} = \frac{E}{20} \)
6. \( \frac{2}{18} = \frac{L}{27} \)
7. \( \frac{6}{N} = \frac{12}{14} \)
8. \( \frac{9}{11} = \frac{O}{44} \)
9. \( \frac{2}{8} = \frac{R}{4} \)
10. \( \frac{5}{V} = \frac{25}{30} \)
11. \( \frac{7}{4} = \frac{Y}{28} \)

Now look for each solution below. Write the corresponding letter on the line above the solution. If you have calculated correctly, the letters will spell her name.

10 9 10 45 49 1 36 7
3 36 6 8 3 10 5 8
A geometric sequence is one in which the ratio between the two terms is constant.

1. **SQUARE NUMBERS** A square number can be modeled by using an area model to create an actual square.

   a. Draw the next two terms in the sequence.

   [Diagram of square numbers]

   1   4

   [Blank spaces for the next two terms]

   b. The function that describes square numbers is \( n^2 \). Complete the table by finding the missing position and the missing value of the term for square numbers.

<table>
<thead>
<tr>
<th>Position</th>
<th>3</th>
<th>11</th>
<th>13</th>
<th>15</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Term</td>
<td>9</td>
<td>64</td>
<td>100</td>
<td></td>
<td>225</td>
</tr>
</tbody>
</table>

2. **TRIANGULAR NUMBERS** A triangular number can be modeled by using symbols to create triangles. The first three triangular numbers are 1, 3, and 6.

   [Diagram of triangular numbers]

   1   3   6

   [Blank spaces for the next two terms]

   a. Draw the next two terms in the sequence.

   b. The function that describes the triangular number sequence is \( n \times \frac{(n + 1)}{2} \). Complete the table for triangular numbers.

<table>
<thead>
<tr>
<th>Position</th>
<th>3</th>
<th>8</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Term</td>
<td>6</td>
<td>10</td>
<td></td>
<td>120</td>
<td>210</td>
<td></td>
</tr>
</tbody>
</table>
Enchanted Rock is a pink granite dome located in Enchanted Rock State Natural Area in Central Texas. It is one of the largest batholiths in the United States. A batholith is made of igneous rock and is the result of volcanic activity. The Enchanted Rock dome rises 425 feet above the ground and is 1,825 feet above sea level.

The entrance fee to Enchanted Rock State Natural Area is $5 per person.

1. Complete the table to find the entrance cost for groups of different sizes.

<table>
<thead>
<tr>
<th>Input, x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, y</td>
<td>$5</td>
<td>$10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write and graph an equation to represent the function displayed in the table. ________________

3. If the park has 290 visitors, how much money did they collect in entrance fees? ________________

4. A local environmental group is planning to hike up Enchanted Rock. The group will cover each member’s entrance fee and will provide lunch for its members. The group budgets $75 for lunch, regardless of the number of people on the hike. Complete the table to show the total expenses of the group based on the number of people on the hike.

<table>
<thead>
<tr>
<th>Input, x</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, y</td>
<td>$100</td>
<td>$125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Write and graph an equation to represent the function displayed in the table. ________________

6. The group will hike up the dome at a rate of 1500 feet per hour. What is their hiking speed per minute? ________________
Stores have sales to attract people to buy their merchandise or to sell off seasonal merchandise at the end of a season. They may advertise 20% off the regular price of an item or $\frac{1}{2}$ off the regular price. Sometimes, stores will offer an extra sale on top of the sale price.

Stores usually advertise the sale price as a percentage or a fraction off the original price. Savvy shoppers know how percentages and fractions compare to know which is a better deal.

**Write a fraction representing how much off the regular price is the store offering.**

1. 25% off all kitchen items!
2. 50% off ELECTRONICS
3. 20% off all outerwear

**Write each fraction as a percent.**

4. Sale Today $\frac{1}{2}$ off
5. $\frac{1}{5}$ off with your Rewards Card
6. $\frac{1}{4}$ off all winter jackets

**Which is the better deal?**

7. Sale Today $\frac{1}{3}$ off all shoes!
   Save 40% on all shoes!
The circle graph below was drawn to show the leading causes of fire in the United States. However, all the labels except one have mysteriously disappeared.

Use the clues below to decide what the labels should be and where they belong. Then complete the graph. (Remember: Each label must include a word or phrase and a percent.)

**Clue 1**  Most fires are caused by *heating equipment*.

**Clue 2**  Fires caused by *electrical wiring* and fires caused by *heating equipment* together make up 46% of all fires.

**Clue 3**  The percent of fires caused by *children playing* is 12% less than the percent of fires caused by *cooking*.

**Clue 4**  The percent of fires caused by *open flames* is equal to the percent of fires caused by *children playing*.

**Clue 5**  The percent of the fires caused by *cooking* and the percent of fires caused by *arson* are together just 1% less than the percent of fires caused by *heating equipment*.

**Clue 6**  The percent of the fires caused by *electrical wiring* is 15% greater than the percent caused by *children playing*.

**Clue 7**  Fires caused by *smoking* and fires caused by *arson* together make up 17% of all fires.

**Clue 8**  Fires that result from other causes are listed in a category called *other*. 
A **percent** is a ratio that compares a number to 100.

\[
\frac{83}{100} = 83 \text{ percent} = 83\% = 0.83
\]

A ratio that compares a number to 1,000 is called a **per mill**. Just like percent, the ratio *per mill* has a special symbol, ‰.

\[
\frac{83}{1000} = 83 \text{ per mill} = 83 \, \text{‰} = 0.083
\]

Throughout the world, the ratio that is used most commonly is percent. However, in some countries, you will find both ratios in use.

**Express per mill as a decimal.**

1. \(325\, \text{‰} \quad 2. \ 71\, \text{‰} \quad 3. \ 6\, \text{‰} \quad 4. \ 900\, \text{‰} \quad 5. \ 20\, \text{‰} \quad 6. \ 100\, \text{‰} \)

**Express each per mill as a fraction in simplest form.**

7. \(47\, \text{‰} \quad 8. \ 400\, \text{‰} \quad 9. \ 100\, \text{‰} \quad 10. \ 25\, \text{‰} \quad 11. \ 150\, \text{‰} \quad 12. \ 30\, \text{‰} \)

**Express each fraction as a per mill.**

13. \(\frac{729}{1000} \quad 14. \ \frac{58}{100} \quad 15. \ \frac{1}{2} \quad 16. \ \frac{3}{4} \quad 17. \ \frac{17}{20} \quad 18. \ \frac{1}{3} \)

**19. CHALLENGE** In the United States, you will sometimes find the **mill** used as a monetary unit. What amount of money do you think is represented by 1 mill?
Solve a simpler problem.
Solve. Use the solve a simpler problem strategy.

1. There are 400 students in Tiffany’s soccer league. Tiffany did a survey of a random sample of 80 students. If 58 of the 80 said they preferred water to juice at halftime, how many students out of 400 would be likely to say the same response?

2. There are 280 families who are members at the Recreation Center. Sheldon conducted a survey of a random sample of 40 families. Of the 40 families, 25 said they would like extended pool hours. Sheldon used the survey to predict that 125 families would prefer extended pool hours. Is his prediction correct? Explain.

3. There are 98 sixth graders at Carter Middle School. A teacher conducted a survey of a random sample of 18 sixth graders to find out how many spend more than one hour a week reading. Of the 18 students, 14 said that they spend more than one hour a week reading. The teacher used the survey to predict that 72 sixth graders at the school spend more than one hour a week reading. Is her prediction correct? Explain.

4. There are 200 members of a chess club. Alfonso conducted a survey of a random sample of 60 members. Of the 60 members, 45 said they would like matches scheduled once a month. Alfonso used the survey to predict that 100 members would like monthly matches. Is his prediction correct? Explain.
Using 100%, 10%, and 1%

Many people think of 100%, 10%, and 1% as key percents.

100% is the whole. 100% of 24 = 1 × 24, or 24.
10% is one tenth of the whole. 10% of 24 = 0.1 × 24, or 2.4.
1% is one hundredth of the whole. 1% of 24 = 0.01 × 24, or 0.24.

Find the percent of each number.

1. 100% of 8,000
2. 10% of 8,000
3. 1% of 8,000
4. 10% of 640
5. 100% of 720
6. 1% of 290
7. 1% of 50
8. 100% of 33
9. 10% of 14
10. 1% of 9

This is how you can use the key percents to make some computations easier.

3% of 610 = ______.
1% of 610 = 6.1,
so 3% of 610 = 3 × 6.1, or 18.3.

5% of 24 = ______.
10% of 24 = 2.4,
so 5% of 24 = $\frac{1}{2}$ of 2.4, or 1.2.

Find the percent of each number.

11. 2% of 140
12. 8% of 2,100
13. 20% of 233
14. 70% of 90
15. 30% of 4,110
16. 5% of 38
17. 50% of 612
18. 25% of 168
19. 2.5% of 320
20. 2.5% of 28
Find the Percent One Number Is of Another

Find the answers, then put the corresponding letter above each answer at the bottom of the page. Not all letters will be used.

T. 12 is what percent of 60? ______
A. What percent of 2 is 8? ______
P. What percent of 800 is 4? ______
M. 16 is what percent of 400? ______
L. What percent of 55 is 33? ______
J. 121 is what percent of 968? ______
H. 36 is what percent of 16? ______
V. What percent of 72 is 27? ______
M. What percent of 40 is 9? ______
A. 52 is what percent of 78? ______
S. 1 is what percent of 250? ______
O. What percent of 64 is 288? ______
R. What percent of 240 is 80? ______
A. 98 is what percent of 140? ______

What is the largest desert in the world?

0.4% 70% 225% 400% 33\%\%\% 66\%\%\%\% 2\%\%\%\%\%\%

How can you determine if an answer will be greater than 100%?
Sales Tax and Discounts

Bargain Betty loves to get a good deal. She will not buy anything unless it is on sale. So, she decided to go to Bobby’s Bargain Basement where everything is on sale. The sales tax rate is 6.5%.

<table>
<thead>
<tr>
<th>Bobby’s Bargain Basement</th>
</tr>
</thead>
<tbody>
<tr>
<td>skirts $26.50 x 40% off</td>
</tr>
<tr>
<td>shirts $24.00 x 25% off</td>
</tr>
<tr>
<td>shoes $28.60 x 15% off</td>
</tr>
<tr>
<td>socks $7.20 x 35% off</td>
</tr>
<tr>
<td>pants $27.40 x 30% off</td>
</tr>
<tr>
<td>coats $34.00 x 45% off</td>
</tr>
</tbody>
</table>

Answer the following questions using Bobby’s price list and find the answers at the bottom of the page. Put the corresponding letter above the answer to find two words describing Betty.

R. What is the discount on shoes?  __________
I. What is the sale price of boots?  __________
R. What is the sale price of a pair of socks?  __________
A. What is the sale price for a pair of pants?  __________
T. What is the discount on shirts?  __________
L. What is the discount on coats?  __________
F. What is the discount on robes?  __________
F. What is the sale price for a sweater?  __________
U. What is the discount on jeans?  __________
T. What is the discount on dresses?  __________
H. What is the total price of a robe including the sales tax?  __________
G. What is the sale price for a belt?  __________
Y. What is the total price of a dress and a skirt including the sales tax?  __________

$25.92  $4.29  $1.85  $11.04  $19.18  $15.30

$6.00  $28.53  $4.68  $26.40  $1.41  $15.95  $33.92
Enrich

Working Backward with Probabilities

Working Backward with Probabilities

Suppose that you are given this information about rolling a number cube.

\[ P(1) = \frac{1}{2} \quad P(3) = \frac{1}{3} \quad P(5) = \frac{1}{6} \]

Can you tell what numbers are marked on the faces of the cube? Work backward. Since a cube has six faces, express each probability as a fraction whose denominator is 6.

\[ P(1) = \frac{3}{6} \quad P(3) = \frac{2}{6} \quad P(5) = \frac{1}{6} \]

So, the cube must have three faces marked with the number 1, two faces marked 3, and one face marked 5.

Each set of probabilities is associated with rolling a number cube. What numbers are marked on the faces of each cube?

1. \( P(2) = \frac{1}{3} \)
   \[ P(4) = \frac{1}{3} \]
   \[ P(6) = \frac{1}{3} \]

2. \( P(1) = \frac{1}{6} \)
   \[ P(4) = \frac{1}{6} \]
   \[ P(2) = \frac{5}{6} \]

3. \( P(1 \text{ or } 2) = \frac{2}{3} \)
   \[ P(2 \text{ or } 3) = \frac{2}{3} \]
   \[ P(1, 2, \text{ or } 3) = 1 \]

Each set of probabilities is associated with the spinner shown at the right. How many sections of each color are there?

4. \( P(\text{red}) = \frac{1}{2} \)
   \[ P(\text{blue}) = \frac{1}{4} \]
   \[ P(\text{green}) = \frac{1}{8} \]
   \[ P(\text{black}) = \frac{1}{8} \]

5. \( P(\text{yellow or purple}) = \frac{5}{8} \)
   \[ P(\text{purple or white}) = \frac{3}{4} \]
   \[ P(\text{green or blue}) = 0 \]
   \[ P(\text{yellow, purple, or white}) = 1 \]
Suppose that you spin the two spinners below. What is the probability that the sum of the numbers you spin is 5?

To find this probability, you first need to count the outcomes. One way to do this is to use a table of sums like the one at the right. From the table, it is easy to see that there are 24 outcomes. It is also easy to see that in 4 of these outcomes, the sum of the numbers is 5. So, the probability that the sum of the numbers is 5 is $\frac{4}{24}$; or $\frac{1}{6}$.

Use the spinners and the table above. Find each probability.

1. $P$(sum is 8) ________  
2. $P$(sum is greater than 6) ________  
3. $P$(sum is 12) ________  
4. $P$(sum is less than or equal to 10) ________

Suppose you toss two number cubes. Each cube is marked with 1, 2, 3, 4, 5, and 6 on its faces. Find each probability. (Hint: On a separate sheet of paper, make a table like the one above.)

5. $P$(sum is a prime number) ________  
6. $P$(sum is a factor of 12) ________  
7. $P$(sum is greater than 12) ________  
8. $P$(sum is less than 6) ________

9. CHALLENGE Here is a set of probabilities associated with two spinners.

$$P(\text{sum is 4}) = \frac{1}{6}, \quad P(\text{sum is 6}) = \frac{1}{3},$$

$$P(\text{sum is 8}) = \frac{1}{3}, \quad P(\text{sum is 10}) = \frac{1}{6}$$

In the space at the right, sketch the two spinners.
People who play games of chance often talk about **odds**. You can find the **odds in favor** of an event by using this formula.

\[
\text{odds in favor} = \frac{\text{number of ways an event can occur}}{\text{number of ways the event cannot occur}}
\]

With the spinner shown at the right, for example, this is how you would find the odds in favor of the event **prime number**.

There are four prime numbers (2, 3, 5, 7).  
\[
\frac{4}{6} = \frac{2}{3}
\]

Six numbers are not prime (1, 4, 6, 8, 9, 10).  
\[
\frac{4}{6} = \frac{2}{3}
\]

The odds in favor of the event **prime number** are \(\frac{2}{3}\) or 2 to 3.

**Suppose that you spin the spinner shown above. Find the odds in favor of each event.**

1. number greater than 3  
2. number less than or equal to 6  
3. even number  
4. odd number  
5. multiple of 3  
6. factor of 10

To find the **odds against** an event, you use this formula.

\[
\text{odds against} = \frac{\text{number of ways an event cannot occur}}{\text{number of ways the event can occur}}
\]

**Suppose that you roll a number cube with 1, 2, 3, 4, 5, and 6 marked on its faces. Find the odds against each event.**

7. number less than 5  
8. number greater than or equal to 2  
9. even number  
10. odd number  
11. number divisible by 3  
12. factor of 12
The Department of Defense headquarters in Washington, D.C. is called the Pentagon. The Pentagon gets its name from the actual shape of the building. It is a regular pentagon, so all of the sides are the same length. The angles in a regular polygon are related in a special way.

1. Use a protractor to measure each angle in the regular polygons below.

2. What do you notice about the measures of the angles in the two triangles?

3. What do you notice about the measures of the angles in the two hexagons?

4. What can you conclude about the angles inside a regular polygon?

5. You can find the measure of an interior angle of a regular polygon with \( n \)-sides by using the formula. \[ m = \frac{(n - 2)(180°)}{n} \] Find the measure of an interior angle of the Pentagon. 

6. If Sabrina builds a pen with 144° interior angles for her turkeys, and all the sides are of equal length, how many sides are on Sabrina’s pen?

7. Draw a regular nonagon. Use a protractor to measure the angles. Use a ruler to measure the sides to make sure that they are equal.
Draw a diagram to solve. Use the blank space at right to draw your diagrams.

1. A window design is made of a rectangle divided by two diagonals. How many sections are there and what are their shapes?

2. Sandra draws a regular hexagon. She divides the hexagon into sections by drawing a line from one vertex of the hexagon to the opposite vertex. How many sections are there and what are their shapes?

3. Harold divides a triangle into sections by drawing a line from one vertex of the triangle to the center of the opposite line. How many sections are there and what are their shapes?

4. A tile is shaped like a square. A design on the tile uses 2 lines to divide the square into sections by connecting the center of one side to the center of the opposite side. How many sections are there and what are their shapes?

5. A student divides a pentagon into sections by drawing a line from one vertex to the center of the opposite line. How many sections are there and what are their shapes?
Enrich

Compass Directions

When a plane is in flight, its direction is expressed as an angle measure. One method of doing this is to give the measure of the angle formed by the plane’s flight path and one of the directions of the compass—north, east, south, or west. For example, this is how you express the two flight paths shown in the figure at the right.

plane A: west 38° north, or W 38° N
plane B: south 72° west, or S 72° W

Write an expression for the direction of each flight path. (You will need to measure the angle with your protractor.)

1.  

2.  

3.  

Use your protractor to draw each flight path.

4.  E 70° S  

5.  E 51° N  

6.  W 75° N
10–4

Enrich

Parallel Lines and Interior Angles

Parallel lines are always the same distance apart and never meet. A line that intersects two parallel lines is called a transversal. A transversal forms angles with the parallel lines that are related.

On the map, Vining Street is parallel to Summer Street. Blueberry Boulevard is a transversal.

The angles between the two parallel lines are called interior angles. Alternate interior angles are on opposite sides of the transversal.

\[ \angle 3 \text{ and } \angle 6 \text{ are alternate interior angles.} \]
\[ \angle 4 \text{ and } \angle 5 \text{ are alternate interior angles.} \]

Alternate interior angles are congruent, so \[ m\angle 3 = m\angle 6 \text{ and } m\angle 4 = m\angle 5. \]

Interior angles on the same side of the transversal are supplementary.

\[ m\angle 4 + m\angle 6 = 180^\circ \]
\[ m\angle 3 + m\angle 5 = 180^\circ \]

You can find the measures of other angles in the diagram by remembering that opposite angles formed by intersecting lines are congruent.

Find the measure of each angle in the figure.

1. \( m\angle 5 \) ________  
2. \( m\angle 1 \) ________  
3. \( m\angle 8 \) ________  
4. \( m\angle 2 \) ________  
5. \( m\angle 7 \) ________  
6. \( m\angle 3 \) ________  
7. \( m\angle 6 \) ________  
8. \( m\angle 4 \) ________
Complete.

1. Define congruent. ________________________________

2. Define similar. ________________________________

3. How many triangles are in the figure? ________________________________

4. Name a triangle congruent to ΔAEF. ________________________________

5. Name a triangle congruent to ΔALK. ________________________________

6. Name three triangles similar to ΔABC. ________________________________

7. Name three triangles similar to ΔAIH. ________________________________

8. How many quadrilaterals are in the figure? ________________________________

9. Name a quadrilateral congruent to JKMB. ________________________________

10. Name one quadrilateral similar to DFIG. What is it? Explain. ________________________________

11. Name a quadrilateral similar to FEHI. ________________________________
African Weaving

For the people of Africa, weaving is a form of art. They have woven intricate and beautiful designs into fabric for many centuries. As with so many other art forms, the beauty of their designs is based on geometric principles.

The designs on this page were created more than 100 years ago in the region of Africa that today is Zaire. They are examples of strip patterns, which were repetitive patterns used as decorative borders on clothing. In the exercises below, you will take a closer look at the geometry of these patterns.

In a strip pattern, the pattern unit is the basic design that is repeated along the strip. For each of these patterns:

a. Identify the pattern unit and make a sketch of it in the space at the right.

b. Name any shapes you recognize that could be used to make the pattern unit.

1.

2.

3.
Enrich

Making Conjectures

A conjecture is an educated guess or an opinion. Mathematicians and scientists often make conjectures when they observe patterns in a collection of data. On this page, you will be asked to make a conjecture about polygons.

Use a protractor to measure the angles of each polygon. Then find the sum of the measures. (Use the quadrilateral at the right as an example.)

1. 

2. 

3. 

4. 

5. Make a conjecture. How is the sum of the angle measures of a polygon related to the number of sides?

6. Test your conjecture. On a clean sheet of paper, use a straightedge to draw a hexagon. What do you guess is the sum of the angle measures? Measure each angle and find the sum. Was your conjecture true?
The word rep-tiles stands for repeating tiles. A geometric figure is a rep-tile if it can be divided into smaller parts according to these rules.

1. All the smaller parts must be congruent to each other.

2. All the smaller parts must be similar to the original tile.

Here are two examples of figures that are rep-tiles.

Divide each rep-tile into four congruent parts.

1. 

2. 

3. 

4. 

5. 

6. 

7. CHALLENGE Show how to use four figures like the one at the right to make a rep-tile.
Suppose an ant is walking across a rectangular grid that is 4 units wide and 7 units long. If the ant walks from one vertex to the opposite vertex in a straight path, the ant will walk across 10 squares of the grid.

For each rectangle, record the width, the length, and the number of squares the ant crosses.

1. 
2. 
3. 
4. 

5. Refer to your answers to Exercises 1–4. What is the pattern?

Now record the width, length, and number of squares the ant crosses for each of these rectangles.

6. 
7. 
8. 

9. Refer to your answers to Exercises 6–8. Does the pattern that you found in Exercise 5 still hold?

10. What is the difference between the rectangles in Exercises 1–4 and the rectangles in Exercises 6–8?

Predict the diagonal path for each rectangle. Check your prediction by sketching a rectangle.

11. 4 units by 9 units 
12. 10 units by 21 units
**Enrich**

You Can Count On It!

**How many triangles are there in the figure at the right?**

**How many parallelograms?**

When counting shapes in a figure like this, you usually have to think of different sizes.

- There are four small triangles.
- There is one large triangle.
- There are five triangles in all.

**You also have to think of different positions.**

- There are three parallelograms in all.

1. Now it’s your turn. How many triangles are in the figure below? How many parallelograms? Use the space at the right to organize your counting.

2. A trapezoid is a quadrilateral with only one pair of sides parallel, as shown at the right. How many trapezoids are in the figure in Exercise 1?
You have learned that area is the number of square units needed to cover a surface. Counting square units on a circular surface can be challenging. Here is a counting method that gives a fairly good estimate of the area of a circle.

Count the squares that cover any part of the circular region. Count the squares that are entirely within the circle.

Find the mean of the two numbers. \( \frac{60 + 32}{2} = \frac{92}{2} = 46 \)
So the area of the circle is about 46 square units.

Estimate the area of each circle or oval.

1. [Diagram of circle]
2. [Diagram of circle]
3. [Diagram of circle]
4. [Diagram of oval]
5. [Diagram of circle]
6. [Diagram of circle]
Enrich

Area of Composite Figures

A composite figure is made up, or composed, of other figures. For example, the L-shaped figure at the right is composed of two rectangles. To find the area of the L-shape, find the area of each rectangle, then add.

<table>
<thead>
<tr>
<th>Area of A</th>
<th>Area of B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = lw )</td>
<td>( A = lw )</td>
</tr>
<tr>
<td>( A = 10 \times 6 )</td>
<td>( A = 20 \times 8 )</td>
</tr>
<tr>
<td>( A = 60 )</td>
<td>( A = 160 )</td>
</tr>
</tbody>
</table>

So the area of the L-shaped figure is \( 60 \text{ ft}^2 + 160 \text{ ft}^2 \), or \( 220 \text{ ft}^2 \).

Find the area of each composite figure.

1. 
   \[
   \begin{array}{c}
   \text{33 in.} \\
   \text{15 in.} \\
   \text{24 in.} \\
   \end{array}
   \]
   \[ \quad \]

2. 
   \[
   \begin{array}{c}
   \text{32 cm} \\
   \text{50 cm} \\
   \text{38 cm} \\
   \text{17 cm} \\
   \end{array}
   \]

3. 
   \[
   \begin{array}{c}
   \text{5 yd} \\
   \text{6 yd} \\
   \text{7 yd} \\
   \end{array}
   \]

4. 
   \[
   \begin{array}{c}
   \text{3 m} \\
   \text{8 m} \\
   \end{array}
   \]

5. **CHALLENGE** Find the area of the shaded region in the figure at the right.
The diagram shows an artist’s designs for a painting of geometric figures. Use data from the diagram to solve the problems. Use \( \pi \approx 3.14 \) round to the nearest whole number.

1. How many square centimeters of the painting is section B?

2. How many square centimeters of the paintings are sections D and H?

3. How many square centimeters of the paintings is sections E?

4. How many square centimeters of the paintings are sections A and F?

5. How many square centimeters of the paintings are sections C and G?

6. How many square centimeters of the paintings are sections D and G?

7. How many square centimeters of the paintings are sections B and E?

8. How many square centimeters of the paintings are sections A, C, and F?
The volume of a three-dimensional figure is the amount of \textit{space} it contains. Volume is measured in cubic units—cubic meters, cubic inches, and so on.

The liquid capacity of a container is the amount of \textit{liquid} it can hold. Liquid capacity generally is measured in units like liters, milliliters, cups, pints, quarts, and gallons.

The chart at the right shows the relationship between volume and liquid capacity. If a container were shaped like the rectangular prism below the chart, this is how you would find its liquid capacity.

<table>
<thead>
<tr>
<th>Volume</th>
<th>Liquid Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = lwh$</td>
<td>$1 \text{ in}^3 \approx 0.544 \text{ fl oz}$</td>
</tr>
<tr>
<td>$V = 7 \times 5 \times 4$</td>
<td>$140 \text{ in}^3 \approx (140 \times 0.544) \text{ fl oz}$</td>
</tr>
<tr>
<td>$V = 140$</td>
<td>$140 \text{ in}^3 \approx 76.16 \text{ fl oz}$</td>
</tr>
</tbody>
</table>

So the liquid capacity of the container is about 76 fluid ounces.

For Exercises 1–4, find the liquid capacity of a container shaped like a rectangular prism with the given dimensions. If necessary, round to the nearest whole number.

1. length, 8 cm  
   width, 4 cm  
   height, 6 cm _______

2. length, 7 ft  
   width, 2 ft  
   height, 3 ft _______

3. length, 4 m  
   width, 2 m  
   height, 5 m _______

4. length, 5 in.  
   width, 1 in.  
   height, 3 in. _______

5. An aquarium is 36 inches long, 18 inches wide, and 18 inches tall. It is filled with water to a height of 12 inches. How many gallons of water are in the aquarium? (Round to the nearest gallon.) _______
A net is a two-dimensional pattern that can be folded to form a three-dimensional figure. For example, the figure at the right is a net for a rectangular prism.

Identify the figure that would be formed by folding each net.

1. 
2. 
3. 
4.

A cube is a rectangular prism in which all the edges have the same length. A net for a cube is made up of six squares. However, not every pattern of six squares is a net for a cube. For example, it would be impossible to fold the pattern at the right to form a cube.

Tell whether each of these patterns is a net for a cube.

5. 
6. 
7. 
8. 
9. 
10. 

11. CHALLENGE In all, there are eleven different patterns of six squares that form a net for a cube. Sketch the eleven patterns in the space below.